The Interaction between Time-nonseparable Preferences and Time Aggregation

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ABSTRACT

In this paper I develop and empirically analyze a continuous-time, linear-quadratic, representative consumer model in which the consumer has time-nonseparable preferences of several forms. Within this framework I show how time aggregation and time nonseparabilities in preferences over consumption streams can interact. I show that the behavior of both seasonally adjusted and unadjusted consumption data is consistent with a model of time-nonseparable preferences in which the consumption goods are durable and in which individuals develop habit over the flow of services from the good. The presence of time nonseparabilities in preferences is important because the data does not support a version of the model that focuses solely upon time aggregation and ignores time nonseparabilities in preferences by making preferences time additive.

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I. Introduction

In this paper I develop and empirically analyze a continuous-time, linear-quadratic, representative consumer model in which the consumer has time-nonseparable preferences of several forms. Within this framework I show how time aggregation and time nonseparabilities in preferences over consumption streams can interact. I show that the behavior of both seasonally adjusted and unadjusted consumption data is consistent with a model of time-nonseparable preferences in which the consumption goods are durable and in which individuals develop habit over the flow of services from the good. The presence of time nonseparabilities in preferences is important because the data does not support a version of the model that focuses solely upon time aggregation and ignores time nonseparabilities in preferences by making preferences time additive.

There has been an extensive amount of work examining whether or not aggregate consumption expenditures are consistent with the restrictions implied by the permanent income hypothesis and consumption-based asset-pricing models. Often these studies find that the theory does not fit the data[1].

Because of these poor empirical results, many other studies relax some of the auxiliary assumptions made in previous studies. A closer examination of the assumption that the decision interval of economic agents matches the interval of data collection caused some authors to replace it with the assumption that the interval between data observations is larger than the actual decision interval. The result is a *temporal aggregation* problem that can often be allowed for in estimation and inference. For example, Christiano, Eichenbaum and Marshall (1987) find that rejections of the

permanent income hypothesis could be due to temporal aggregation problems.[2]

Concurrently, there has been much work examining the impact of time-nonseparable preference specifications upon the consumption-based CAPM and the permanent income hypothesis. Dunn and Singleton (1985), Eichenbaum and Hansen (1990) and Ogaki (1988), for example, empirically examine discrete-time models that allow for durability in the consumption data. Although allowing for durability seems to help the fit of the model, the results do not provide a complete rationalization of consumption and asset price data.

The purpose of this paper is to show how time-nonseparable preference specifications along with time aggregation can help us to understand much about the behavior of aggregate consumption. One motivation for such a study is the fact that all studies, which examine the empirical relevance of the temporal aggregation problem, conduct the analysis within a framework of time-separable preferences. Within a continuous-time or fine discrete-time model, this assumption is far from appealing since it implies that an individual's preferences over consumption at one instant are unaffected by consumption the instant before. In this vein, Huang and Kreps (1987) and Hindy and Huang (1989) argue that the assumption of time-separability, within a continuous-time model, should be dropped. These authors also argue that preferences should be specified such that consumption at dates near time t should be relatively substitutable for consumption at time t. The argument against using a separable preference specification can also be made on the grounds of a measurement problem: we have observations on consumption that for most National Income and Produce Account classifications have a very durable nature. For example, expenditures on clothing are defined as being nondurable.

In conducting this study, I develop a continuous-time linear-quadratic permanent income model augmented with a general form of time nonseparabilities in preferences. The model assumes a very simple technology with a fixed rate of return to investment. Although this is a very severe restriction, my goal is to use this model as a laboratory for understanding how temporal aggregation and nonseparabilities interact and for determining what type of preference specifications should or should not be ruled out before moving on to more complicated environments.

I apply several examples of the model to seasonally adjusted and unadjusted nondurable plus services consumption data. I show that the time-additive version of the model is not consistent with the seasonally adjusted consumption data at monthly frequencies. However, the model is consistent with the data if the consumption goods are allowed to be durable. Further I show that there is some weak evidence for habit persistence (of the form studied by Ryder and Heal (1973), Sundaresan (1989) and Constantinides (1990), for example) <u>only</u> if the habit is developed over the flow of services from the durable nature of the goods.

Using quarterly seasonally unadjusted data I present evidence that the durable nature of the goods may be understated in seasonally adjusted data and that time-separable models perform <u>very poorly</u>. Also this data indicates that there is a significant role for habit persistence in understanding the seasonal behavior of consumption.

Along with these general empirical points I also address the following questions: Can the presence of time-nonseparabilities mitigate the effects of temporal aggregation and make a discrete-time, time-separable model relatively consistent with the data? Can temporal aggregation overturn the negative autocorrelation in consumption that is usually introduced with

durable goods, and make a durability story consistent with the positive correlation in quarterly consumption?

The rest of the paper is structured as follows. In Section 2, I examine a time-additive model that allows me to focus upon the effects of temporal aggregation. I show that although the model is consistent with quarterly seasonally adjusted data the model is very inconsistent with monthly seasonally adjusted data and quarterly seasonally unadjusted data.

In Section 3, I develop a model with very general forms of time nonseparabilities in preferences. I show that with proper restrictions on preferences, a law of motion for consumption can be derived that can be used to estimate the preference parameters. I also discuss why it is necessary to focus upon specific parametric forms of the preferences.

Section 4 examines the capital stock and consumption dynamics implied by the model in the case of habit persistence, in a world of perfect certainty. I discuss further restrictions that are necessary so that the model produces reasonable economic results.

In Section 5, I empirically examine several examples of the time-nonseparable preference structure of Section 3, using seasonally adjusted consumption data. First, I examine the effect of making preferences time-nonseparable by assuming that consumption goods are durable with exponential depreciation. I also examine some specifications that allow for habit persistence effects. Section 6 empirically examines a version of the model that allows for seasonality in consumption and takes this model to seasonally unadjusted data. Section 7 concludes the paper.

II. A Time-Separable Model

In this section, I examine a continuous-time linear-quadratic permanent income model. The preferences of the representative consumer in the model of this section are assumed to be *time-separable* so that I can first focus upon the time aggregation problem. The model is a continuous-time version of a model studied by Hansen (1987) and Sargent (1987). Within the context of this model, I show, as did Christian, Eichenbaum and Marshall (1989), that time aggregation can explain much of the behavior of quarterly seasonally-adjusted (s.a.) aggregate consumption. However, I present evidence using monthly s.a. consumption and quarterly seasonally unadjusted (s.u.) consumption data that is inconsistent with the model and suggests that time aggregation does not provide a complete rationalization of the data[3]. This motivates the need to look at time-nonseparable specifications.

II.A. A Continuous-Time Permanent Income Model

Assume first that there is a representative consumer with preferences over consumption streams, $c^* = \{c^*(t): 0 \leq t < \infty\}$, of the form:

$$(2.2) \qquad U^*(c^*) = -(1/2)\, E\{\int_0^\infty exp(-\rho^* t)\ (c^*(t) - b^*(t))^2 dt\}$$

where b(t) is assumed to grow at the rate μ:

$$(2.3) \qquad b^*(t) = exp(\mu t)b,\ \mu > 0.$$

The geometric trend in $b^*(t)$ will induce growth in the consumption process

chosen by the consumer. Consider *detrended consumption*, $c = \{c(t): 0 \le t < \infty\}$, which is given by:

$$(2.4) \qquad c(t) = exp(-\mu t)c^{*}(t), \quad t \ge 0.$$

Then the preferences of the consumer over trending consumption given in (2.1) can be rewritten as preferences over detrended consumption:

$$(2.5) \qquad U(c) = -(1/2)\, E(\int_{0}^{\infty} exp(-\rho t)\ (c(t) - b)^{2} dt),$$

where $\rho = \rho^{*} + 2\mu$. In what follows, I will refer to detrended consumption as *consumption*. As I will show subsequently, this detrending does not remove all of the nonstationarity in consumption. In fact, the detrended consumption process, c, will have a unit root. This is to be contrasted with the model of Christiano, Eichenbaum and Marshall (1987) in which detrended consumption is stationary.

There is a technology for transferring consumption over time that has a constant rate of return of ρ:

$$(2.6) \qquad Dk(t) = \rho k(t) + e(t) - c(t), \quad t \ge 0, \ k(0) \text{ given},$$

where $\{e(t): 0 \le t < \infty\}$ is stochastic process that describes the behavior of endowments in the economy and $k(t)$ gives the level of the capital stock in the economy at time t. The random endowment provides the source of uncertainty in the economy. Note that I have equated the risk free rate of return to the discount factor *after detrending* the consumption process. This corresponds to the assumption made by Flavin (1981), for example.

Christiano, Eichenbaum and Marshall (1987) follow a different route and equate the discount factor to the rate of return, *before* detrending.

The consumer maximizes the objective function (2.3), subject to the constraint (2.6)[4]. In Section 3, I consider solutions to a general model that nests this model, however it is easy to see that the optimal consumption process will be a martingale just as in Hall's (1978) model[5]. This occurs because a unit of the consumption good given up at period t yields $exp(\rho\tau)$ units at period $t+\tau$, via (2.6). As a result, the consumer equates the marginal utility of the consumption good (*muc*) today to discounted expected marginal utility of consumption tomorrow, times $exp(\rho\tau)$:

$$(2.7) \qquad muc(t) = exp(\rho\tau)E(exp(-\rho\tau)muc(t+1)|\mathcal{F}(t)), \ \tau>0$$
$$= E(muc(t+\tau)|\mathcal{F}(t)).$$

where $\mathcal{F}(t)$ gives the information set at time t. Since the marginal utility of consumption at time t is $-(c(t) - b)$, (2.7) implies that:

$$(2.8) \qquad c(t) = E(c(t+\tau)|\mathcal{F}(t)), \ \tau > 0.$$

The time derivative of consumption then satisfies:

$$(2.9) \qquad Dc(t) = D\xi(t)$$

where $D\xi(t)$ is the (instantaneous) innovation in the processes describing the permanent value of the endowment shock process at time t. I assume that the (random) measure induced by $D\xi(t)$ satisfies $E(D\xi(t)^2) = \sigma^2 dt$.

II.B. *Time Series Implications for Time-Averaged Consumption*

The data are observations of time-averaged consumption expenditure over a unit of time. Although consumption is a martingale in continuous time, differences in time-averaged consumption[6] will not be a martingale in discrete time, when sampled at the integers. To see this, let $\bar{c}(t) = \int_{t-1}^{t} c(\tau)d\tau$. Then consider:

$$
\begin{aligned}
(2.10)\qquad \bar{c}(t+1) - \bar{c}(t) &= \int_{t}^{t+1} c(\tau)d\tau - \int_{t-1}^{t} c(\tau)d\tau \\
&= \int_{t}^{t+1} [c(\tau) - c(\tau-1)]d\tau \\
&= \int_{t}^{t+1} \left[\int_{0}^{1} D\xi(\tau-1+r)dr\right]d\tau .
\end{aligned}
$$

By changing the order of integration in the last term of (2.10), $\bar{c}(t+1) - \bar{c}(t)$ can be expressed as:

$$(2.11)\qquad \bar{c}(t+1) - \bar{c}(t) = u(t+1),$$

where $\{u(t): t=0,1,2,\ldots\}$ has the first-order moving-average representation:

$$(2.12)\qquad u(t) = \int_{t-1}^{t} (t-\tau)D\xi(\tau) + \int_{t-2}^{t-1} (\tau-t+2)D\xi(\tau), \quad t=0,1,2,\ldots .$$ [7]

The first-order autocorrelation for u is:

$$(2.13)\qquad R(1) = \frac{E(u(t)u(t-1))}{E(u(t)^2)} = \frac{(1/6)\ \sigma^2}{(2/3)\ \sigma^2} = 0.25 .$$

This result was originally derived by Working (1960).

In a discrete-time version of this model, where the observed consumption data is matched with the consumption process in the model, consumption differences are predicted to be uncorrelated over time [see e.g. Hall (1978)]. The fact that the first difference in time-averaged consumption is correlated at the first lag, could help to explain some of the rejections of the discrete-time permanent income hypothesis. This is what Christiano, Eichenbaum and Marshall (1987) find when they use quarterly data.

II.C Data

To test the implications of the models in this paper, I focused solely upon consumption data[8]. The consumption measure I used was per capita real expenditures on nondurables and services where the aggregate consumption expenditures are those measured by the U.S. Department of Commerce. Both seasonally adjusted (s.a.) and seasonally unadjusted (s.u.) quarterly data was used. The seasonally adjusted quarterly data runs from the first quarter of 1952 to the end of 1986. The seasonally unadjusted data runs from the first quarter of 1959 to the end of 1986[9]. Monthly seasonally adjusted data from January 1959 to the end of 1986 was also used.

II.D Tests of the Model Using Quarterly and Monthly, S.A. Consumption Data

The model predicts that $\bar{c}(t) - \bar{c}(t-1)$ is a moving average process of order 1. Parameterize this moving average as:

$$(2.14) \quad \bar{c}(t) - \bar{c}(t-1) = \theta_0\epsilon(t) + \theta_1\epsilon(t-1),$$

where $E(\epsilon(t)^2) = 1$ and $E(\epsilon(t)\epsilon(\tau)) = 0$ for $\tau \neq t$. Under the assumption that R(1) is 0.25, we can estimate θ_0 and the trend parameter μ. The parameter θ_1 is then implied by the autocorrelation restriction. Table 2.1 gives estimates of the parameters of the model with R(1) restricted to 0.25 using seasonally-adjusted consumption. Estimates of (2.14) are given with θ_2 unrestricted are given in Table 2.2. In both tables estimated parameters are reported for the period 1952,1 to 1986,4 and 1959,1 to 1986,4. The latter subsample was used since this matches the period of the monthly data. Log-likelihood values are also reported in each table. Notice that the log-likelihood for each sample marginally improves from Table 2.1 to Table 2.2. Likelihood ratio tests of the 0.25 restriction for R(1) yield probability values of 0.73 for that data set from 1952,1 to 1986,4 and 0.78 for data from 1959,1 to 1986,4. This occurs because the first-order autocorrelation of differences in detrended consumption[10] was estimated to be 0.276 (0.067)[11] for the longer data set and 0.260 (0.081) for the shorter data set. Hence the restriction on R(1) implied by the model is very consistent with the data.

The model also implies that information lagged two periods should not be useful in predicting the consumption change today. Table 2.3 reports likelihood ratio tests and P-values of tests of the time additive model against higher order moving-average models for detrended consumption differences. For the longer data set, tests of the model against moving-average models of orders 2, 3 and 4 do not reject at the 5% significance level. The moving average model of order 5 for the longer data set and the moving average model of orders 4 and 5 for the shorter data set

provide more evidence against the model. However, this could be due to spurious correlation induced through seasonal adjustment or to the presence of seasonal patterns that are not completely removed.[12] Other than these cases, the tests in Table 2.3 indicate that the time-additive model is reasonably consistent with the data as noted by Christiano, Eichenbaum and Marshall (1987).

Now consider <u>monthly</u> measures of seasonally adjusted consumption. Since consumption is a martingale in continuous time, the nature of the results should be robust to different intervals of time aggregation. However, the first-order autocorrelation in first-differences of detrended monthly consumption is estimated to be -0.188 (0.052), which is significantly <u>negative</u>. The trend was estimated using maximum likelihood just as was done for the quarterly data. Table 2.4 reports estimates of the trend and the moving average parameters for the difference in consumption with and without the 0.25 restriction using monthly seasonally adjusted data. A likelihood ratio test of the restriction yields a probability value of essentially zero. As a result, the model does not seem to be very consistent with monthly seasonally adjusted data.

Why is this happening? Why is the model successful at quarterly frequencies but not at monthly frequencies? In the next section I relax the time-separability assumption imposed upon preferences by (2.3) and show that the negative first-order correlation in monthly data is not surprising. Before moving on to this preference-based explanation for the changing autocorrelation values, another explanation for this finding must be examined. Consider a discrete-time version of this time separable model in which the model is assumed to be correct at monthly frequencies. In this case, the model implies that monthly consumption is a martingale. Suppose

now that observed consumption is equal to the model consumption plus an i.i.d. measurement error that is also independent of the model shocks. Then observed consumption differences are given by:[13]

$$(2.15) \qquad \bar{c}(t) - \bar{c}(t-1) = u_1(t) + u_2(t) - u_2(t-1)$$

where u_1 is the model error and u_2 is the i.i.d. measurement error.

Let $E(u_1(t)^2) = \sigma_1^2$ and $E(u_2(t)^2) = \sigma_2^2$. Note that (2.15) implies that consumption differences have a first-order moving average representation and that the first-order autocorrelation value is $R(1) = -\sigma_2^2/(\sigma_1^2 + \sigma_2^2)$. If we let $\sigma_1^2 = 1$, then the first-order autocorrelation for detrended monthly consumption differences gives us an estimate of σ_2^2 of 0.232.

Given the behavior of consumption at monthly frequencies in (2.15), it is easy to show that quarterly consumption also follows a first-order moving average process where the first-order autocorrelation value is given by: $R_q(1) = (19\sigma_1^2 + 6\sigma_2^2)/(4\sigma_1^2 + 6\sigma_2^2) = (19 + 6\sigma_2^2)/(4 + 6\sigma_2^2)$. Using the estimated value of σ_2^2 from the monthly data, this implies an estimate of $R_q(1)$ of 0.162. Although this is not equal to the value of 0.276 found with quarterly data from 1959 to 1986, a likelihood ratio test of whether $R_q(1)$ is equal to 0.162, does not reject this restriction at the 10% significance level. As a result, this measurement error model, along with temporal aggregation, is capable of explaining the changing first-order autocorrelation values in quarterly and monthly data. However, is an i.i.d. measurement error model for consumption levels reasonable?

As Wilcox (1988) has argued, a component of the measurement error in consumption is due to the misallocation of retail sales into the different categories of consumption by the Commerce Department. Since the allocation

of retail sales into consumption classifications is based upon periodic surveys of retail establishments, this introduces substantial *positive* correlation in the measurement error. If any other component of the measurement error is due to the collection practices of the Department of Commerce, and if these practices do not change very often, this will also introduce positive autocorrelation in the measurement error. Any positive autocorrelation in measurement error will reduce the role of measurement error in explaining the behavior of consumption at monthly and quarterly frequencies and will leave a large role for the economic model that I consider in the next sections[14]

II.E Tests of the Model Using Quarterly S.U. Consumption Data

In this subsection I examine the implications that this simple model has for seasonally unadjusted data. To account for seasonality in the data, the model must be modified in some manner. One way to induce seasonality in consumption is to add a seasonal deterministic bliss point process that will induce a deterministic seasonal pattern in consumption. Seasonal bliss point movements have been used in a similar way by Miron (1982). In Section 6 I show that the time additive model with a deterministic bliss point process implies that consumption differences follow the process:

$$\dot{c}(t) - \dot{c}(t-1) = u(t) + B(t) \tag{2.16}$$

where again $u(t)$ has an MA(1) representation with first-order autocorrelation equal to 0.25, and $B(t)$ is a sequence of deterministic seasonal dummies.

As a first pass, consider the autocorrelation structure of the first differences in s.u. consumption with trend and seasonal dummies removed. Table 2.5 gives the autocorrelation values for residuals. The trend value and seasonal dummies were removed using a likelihood function in which the residuals were assumed to be white noise. Although this may be a misspecified likelihood function, it will yield consistent estimates of the autocorrelation values.

There are two important things to notice. First, unlike the seasonally adjusted quarterly data, the first-order autocorrelation is not close to 0.25. Hence with this different manner of accounting for seasonality, the time-additive model is <u>not</u> <u>consistent</u> with the data even at Quarterly frequencies. Second, note that the autocorrelation value at the fourth lag is significantly positive and large. This indicates that the use of seasonal dummies does not remove all of the seasonality in the data and some addition must be made to the model to account for this behavior.

III. A Model with Time-Nonseparable Preferences

In this section, I modify the model of section 2 and introduce temporal dependencies in preferences over consumption. Nonseparabilities are introduced by specifying a mapping from current and past consumption goods into a process called *services*. The representative consumer is assumed to have time-separable preferences over services. The mapping from consumption into services is a type of Gorman-Lancaster[15] technology in which the consumption goods are viewed as bundled claims to characteristics that the consumer cares about[16]. The form of nonseparabilities nests models of durable goods and habit as special parametric examples. I provide

sufficient conditions on the mapping from consumption to services such that the consumer's preferences and the capital accumulation problem are well defined.

An important result that is exploited in Sections 4, 5 and 6 is that under these assumptions, the marginal utility process is a martingale. Using this fact, a univariate representation for observed consumption is derived. I also show that to give this model interesting empirical content, *a priori* restrictions must be placed upon the form of the nonseparabilities. This provides motivation for looking at specific forms of nonseparabilities in sections 4, 5 and 6.

III.A. A Commodity Space for Services

The preferences of the consumer in this model will be defined over processes that I call *services*. In the construction of the commodity space for services, the first piece that is needed is an information structure for the economy.

Let $(\Omega, \mathcal{F}, Pr)$ be the underlying probability space. Information in the economy is represented by a sequence of sub sigma-algebras of $\mathcal{F}$ (a *filtration*): $F \equiv \{\mathcal{F}(t): t \in [0,\infty)\}$. $\mathcal{F}(t)$ gives the set of distinguishable events at period t. I assume that $\mathcal{F}(t) \subseteq \mathcal{F}(s)$ for $t \leq s$, so that information is not lost over time. Let $\Omega^+ \equiv \mathbb{R}^1_+ \times \Omega$, then a stochastic process is a function, $x:\Omega^+ \rightarrow \mathbb{R}$, where $x(t)$ is the value of the process at time t. Two spaces of stochastic processes will be important.

Let $\mathcal{F}^+$ be the product sigma-algebra given by $\mathcal{F} \times \mathcal{B}^1_+$ where $\mathcal{B}^1_+$ denotes the Borel sets of $\mathbb{R}^1_+$. Let λ^* be a measure on $\mathbb{R}^1_+$ that has density $exp(-\rho^* t)$ with respect to Lebesgue measure, where $\rho^* > 0$. I denote the product measure

given by $Pr\times\lambda^*$, as Pr^*. The first space of processes that is needed is the space of square-integrable processes: $L^2(\Omega^+,\mathcal{F}^+,Pr^*)$.

The second space of processes is used to limit the choice space of the representative consumer and is the commodity space for services. Let $\mathcal{P}$ be the predictable sigma-algebra[17] of subsets of Ω^+. Services are required to be measurable with respect to $\mathcal{P}$ and square integrable, i.e., an element of $\mathcal{L}^2(\Omega^+,\mathcal{P},Pr^*)$. Since the consumer will be making choices over services, requiring them to be predictable implies that, at t, the consumer can use only information generated by $\{\mathcal{F}_s : s < t\}$.

III.B. Preferences over Services

The preferences of the representative consumer are assumed to be time-nonseparable over consumption. However it is convenient to first assume that the consumer's preferences are time-separable over a stochastic process, $s^*=\{s^*(t):0\le t<\infty\}$, called *services*. The service process is assumed to lie within the space $\mathcal{L}^2(\Omega^+,\mathcal{P},Pr^*)$. Given a member of this space, s^*, the representative consumer evaluates s^* via the utility function:

$$(3.1) \qquad U(s^*) = -(1/2)E\left(\int_0^\infty exp(-\rho^* t)(s^*(t)-b^*(t))^2 dt\right)$$

where $\{b^*(t):0\le t<\infty\}$ is a deterministic process describing the bliss point movement. To model growth, I assume that $b^*(t)$ grows at the rate μ:

$$(3.2) \qquad b^*(t) = exp(\mu t)b(t),\ \mu > 0.$$

When b(t) is a constant, the geometric trend in $b^*(t)$ induces growth in the

service process chosen by the consumer and, as a result, growth in the consumption process. Consider now *detrended services*, $s = \{s(t):0 \le t < \infty\}$ given by:

$$(3.3) \qquad s(t) = exp(-\mu t)s^*(t), \ t \ge 0.$$

Then the preferences of the consumer over trending services given in (3.1) can be rewritten as preferences over detrended services:

$$(3.4) \qquad U(s) = -(1/2)E\{\int_0^\infty exp(-\rho t)[s(t)-b(t)]^2 dt\}$$

where $\rho = \rho^* + 2\mu$. In what follows, I will refer to detrended services as *services*. Also, I will refer to *detrended consumption* as *consumption*, where consumption is detrended in the same manner as in (3.3). Let λ be a measure on $\mathbb{R}^1_+$ that has density $exp(-\rho t)$ with respect to Lebesgue measure and let $Pr^+ = Pr \times \lambda$. Note that detrended services lie within $\mathcal{L}^2(\Omega^+, \mathcal{P}, Pr^+)$.

III. C. Time Nonseparabilities

Time nonseparability in the consumer's preferences over consumption is introduced by making $s(t)$ a linear function current and past consumption. The dependence of services on current and past consumption will be modeled by making services at time t a convolution between a nonrandom distribution on $\mathbb{R}^1_+$, g, and consumption, c:

$$(3.5) \qquad s^1 = g \star c$$

where $\star$ denotes convolution of two distributions. The distribution g will often put weight on all of $\mathbb{R}^1$, in which case, (3.5) is defined by setting c to be zero for $\tau < 0$. One way to think of this convolutions is to let $s^1(t)$ be given by:

$$(3.6) \qquad s^1(t) = \int_0^t g(\tau)c(t-\tau)d\tau.$$

However, the integral in (3.6) will, not in general be given by standard notions of integration.

s^1 is a process that gives the contribution to services from consumption purchases from time 0 onwards. In order to allow for initial conditions for the service process, let $s^2(t)$ be a nonrandom member of $\mathcal{L}^2(\Omega^+,\mathcal{P},Pr^+)$. $s^2(t)$ gives the contribution to services at time t from the initial stock of services. The service process at time t is then given by:

$$(3.7) \qquad s(t) = s^1(t) + s^2(t), \ t\geq 0.$$

Although s^2 could be collapsed into the bliss point process, b, it will be useful to carry along a separate process for initial conditions.

For a particular g, (3.4), (3.5) and (3.7) give the consumer's preferences over consumption. Below I show how to restrict g and the space of consumption processes so that the result in (3.7) is in $\mathcal{L}^2(\Omega^+,\mathcal{P},Pr^+)$.

Examples of g

1. Exponential Depreciation of a Durable Good. Let $s(t)$ be the service process generated by a durable that undergoes exponential

depreciation. In this case, g is given by:[18]

$$(3.8)\qquad g(t) = \begin{cases} 0 \text{ if } t < 0 \\ exp(\delta t)\ t \geq 0 \end{cases}, \text{ where } \delta < \rho/2.$$

Suppose that $c_a(t) = \int_0^t c(\tau)d\tau$ is a diffusion process. Then the first service process is given simply by:

$$(3.9)\qquad s^1(t) = \int_{[0,\infty)} exp(\delta\tau)Dc_a(t-\tau)d\tau$$

where the integral in (3.9) is defined as a stochastic integral in the usual way. Suppose further that the initial condition is given by a stock of services in the amount $\mathcal{J}(0)$ at $t=0$. Then the second service process is given by:

$$(3.10)\qquad s^2(t) = exp(\delta t)\mathcal{J}(0).$$

In fact, this initial condition could be collapsed into the consumption process by adding to it a mass at 0 in the amount $\mathcal{J}(0)$. The service process can be given succinctly as a solution to the stochastic differential equation:

$$(3.11)\qquad Ds(t) = \delta s(t) + Dc_a(t), \text{ with } s(0) = \mathcal{J}(0).$$

2. Habit Persistence. Suppose that the consumer cares about the level of consumption today relative to an average of past consumption so that the consumer develops an acceptable level of consumption over time. *Habit Persistence* of this form has been studied, for example, by Constantinides

(1990), Detemple and Zapatero (1989), Novales (1990), Pollack (1970), Ryder and Heal (1973), and Sundaresan (1989). Following Constantinides (1990) I model habit persistence by assuming that $s^1(t)$ is of the form:

$$(3.12) \qquad s^1(t) = c(t) - \alpha(-\gamma)\int_{(0,\infty)} exp(\gamma\tau)c(t-\tau)d\tau, \ \gamma<0, \ 0<\alpha<1$$

Note that the term $(-\gamma)\int_0^\infty exp(\gamma\tau)c(t-\tau)d\tau$ is a weighted average of past consumption, and α gives the proportion of this average that is compared to current consumption to arrive at the level of services today. The process $x(t) = (-\gamma)\int_0^\infty exp(\gamma\tau)c(t-\tau)d\tau$ is referred to as the *habit stock*.

In this example, g is then given by:

$$(3.13) \qquad g = \Delta - \alpha\eta$$

where Δ is the dirac delta function and η is given by:

$$(3.14) \qquad \eta(t) = \begin{cases} 0 \text{ if } t \le 0 \\ exp(\gamma t) \ \ t > 0 \end{cases} .$$

The initial condition for services are given by:

$$(3.15) \qquad s^2(t) = -\alpha exp(\gamma t)x(0)$$

where $x(0)$ is the initial value of the habit stock.

Many other examples of time-nonseparable preferences can be mapped into this notation.

III. D. Restrictions on g and c

Since the preferences of the representative consumer are defined over elements of $\mathcal{L}^2(\Omega^+,\mathcal{P},Pr^+)$, some restriction needs to be placed upon the space of consumption processes and the distribution g such that the convolution given by (3.5) results in a member of $\mathcal{L}^2(\Omega^+,\mathcal{P},Pr^+)$. Often it will be desirable that the mapping be from all of $\mathcal{L}^2(\Omega^+,\mathcal{P},Pr^+)$ into $\mathcal{L}^2(\Omega^+,\mathcal{P},Pr^+)$ as in the time-additive and habit persistence models.

The first restriction that I impose insures that the mapping yields a member of $\mathcal{L}^2(\Omega^+,\mathcal{F}^+,Pr^+)$. To describe this restriction, define a *discounted* version of g, denoted g′:

$$(3.16) \qquad g' = gh$$

where $h(t) = exp(-\epsilon t)$ for $t \geq 0$ and zero otherwise and $\epsilon = \rho/2$. Similarly let $c' = ch$, $(Dc)' = Dch$ and so on. The first restriction is in the form of a restriction upon g and the space of admissible consumption processes:

Assumption 1: The space of admissible consumption processes is restricted to be:

$$(3.17) \qquad C^{\ell} = \{D^{\ell}c \in \mathcal{L}^2(\Omega^+,\mathcal{P},Pr^+)\}$$

where $\ell \geq 0$ and ℓ is the smallest integer such that $(\epsilon^2+\omega^2)^{-\ell}\,\hat{g}'(\omega)\hat{g}'(\omega)^*$ is essentially bounded and where $\hat{g}'$ is the fourier transform of g′.

To see that Assumption 1 implies that (3.5) maps the space of admissible

consumption processes into $\mathcal{L}^2(\Omega^+,\mathcal{F}^+,Pr^+)$, consider:

$$(3.18)\qquad E(\int_0^\infty exp(-\rho t)[\ g{\star}c(t)\ ]^2dt) = E(\int_0^\infty [g'{\star}c(t)]^2dt)$$
$$= E(\int_{-\infty}^{+\infty} \hat{g}'(\omega)\hat{g}'(\omega)^{*}\ \widehat{c'}(\omega)\ \widehat{c'}(\omega)^{*}d\omega).$$

where $\widehat{c'}(\omega)$ is the fourier transform of c'. The second equality in (3.18) follows from the Parseval formula. Note that $(\epsilon + D)(D^jc)'(t) = (D^{j+1}c)'$. Hence we have:

$$(3.19)\qquad E(\int_0^\infty exp(-\rho t)[\ g{\star}c(t)\ ]^2dt)$$
$$= E(\int_{-\infty}^{+\infty} (\epsilon^2+\omega^2)^{-\ell}\ \hat{g}'(\omega)\hat{g}'(\overset{*}{\omega})\widehat{(D^{\ell}c)'}(\omega)\widehat{(Dc^{\ell})'}(\omega)^{*}d\omega.$$

Assumption 1 and the Hölder inequality imply that (3.19) is finite.

The consumption process is assumed to be compatible with the information process in the economy. I will need to further restrict the distribution g so that the mapping (3.5) does not destroy the information structure in the economy:

Assumption 2: The Laplace transform of g, $\tilde{g}(\zeta)$, is analytic in the half plane: $\{\zeta:\mathrm{Re}(\zeta) > \rho/2\}$. Further, for each real $\sigma' > \rho/2$ and $\zeta = \sigma + i\omega$ where $\sigma > \sigma'$, $|\tilde{g}(\zeta)| \leq |\mathcal{P}_K(\zeta)|$ for some polynomial $\mathcal{P}_K$ which depends on compact sets $K\subset[\sigma',\infty)$ where $\sigma\in K$.

Using Theorem 2.5 of Beltrami and Wohlers (1966, p. 51), Assumptions 1 and 2, imply that g is one-sided,[19] putting weight only on $\mathbb{R}^1_+$ and maps (possibly a subset of) $\mathcal{L}^2(\Omega^+,\mathcal{P},Pr^+)$ into $\mathcal{L}^2(\Omega^+,\mathcal{P},Pr^+)$.

III. E. *Capital Accumulation*

The model is completed by assuming that there is a capital accumulation technology that allows for the transfer of consumption over time at the constant rate ρ:

$$(3.20) \quad Dk(t) = \rho k(t) + e(t) - c(t), \ k(0) \text{ given,}$$

where $k(t)$ is the level of the capital stock at period t, $k(0)$ is an initial condition and $e(t)$ is an endowment process of the consumption good. If e is an element of $\mathcal{L}^2(\Omega^+,\mathcal{P},Pr^+)$ and if c satisfies Assumption 1, then $k(t)$ is an element of $\mathcal{L}^2(\Omega^+,\mathcal{P},Pr^+)$.

The problem facing the consumer is to maximize the objective function (3.1) subject to the mapping (3.5) and the capital technology (3.20) by choice of $D^{\ell}c(t)$. Because the control variable of the problem is not necessarily consumption itself but is a derivative of c, it is convenient to rewrite the mapping (3.5) as:

$$(3.21) \quad s^1 = g_{\ell} * D^{\ell}c.$$

The distribution g_{ℓ} can always be found using a result that is analogous to integration by parts[20].

The representative consumer solves the following resource allocation problem:

(MP) Max $(-1/2)E\{\int_0^\infty exp(-\rho t)[s(t)-b(t)]^2dt\}$

subject to: $s(t) = g_\ell * D^\ell c + s^2(t)$ and

$Dk(t) = \rho k(t) + e(t) - c(t)$, $k(0)$ given

by choice of a $D^\ell c$ process in C^ℓ. Letting $y_j(t) = D^j c$ for j=0,2, ... ℓ, the Lagrangian for this problem is given by:

$$
\begin{aligned}
\mathcal{L} = -(1/2)\ E\Big\{\int_0^\infty exp(-\rho t)\Big[& \ [g_\ell * y_\ell(t) + s^2(t) - b(t)]^2 \\
& - \lambda_k(t)\Big[k(t) - k(0) - \rho\int_0^t k(r)dr - \int_0^t e(r)dr + \int_0^t y_0(r)dr\Big] \\
& - \sum_{j=0}^{\ell-1} \lambda_j(t)\Big[y_j(t) - y_j(0) - \int_0^t y_{j+1}(r)dr\Big]\Big]\Big\}
\end{aligned}
$$

where λ_k and λ_j, j=0,1, ..., ℓ-1 are lagrange multipliers.

To characterize the first-order conditions for this optimization problem let g_ℓ^f be a distribution constructed in the following manner. Let g_ℓ^- be a distribution that, assigns weight $g_\ell(t)$ to -t so that g_ℓ^- assigns weight only to the nonpositive real numbers. Let h be the function given by:

$$
(3.22) \qquad \bar{h} = \begin{cases} exp(-\rho t) \text{ if } t \le 0 \\ 0 \text{ otherwise} \end{cases}.
$$

Then g_ℓ^f is given by $g_\ell^- \bar{h}$.

The first-order conditions for the choice of $k(t)$ and y_j, j=0,1, ... ℓ for each t≥0, are then given by:

$$(3.23) \quad E\left\{-g_\ell^f*[g_\ell*y_0(t) + s^2(t) - b(t)]\,|\,\mathcal{F}(t)\right\} - E\left\{\int_0^\infty exp(-\rho t)\lambda_{\ell-1}(t+\tau)d\tau\,|\,\mathcal{F}(t)\right\} = 0,$$

$$(3.24) \quad \lambda_j - E\left\{\int_0^\infty exp(-\rho\tau)\lambda_{j-1}(t+\tau)d\tau\,|\,\mathcal{F}(t)\right\} = 0,\ j = 1,2,\ \ldots\ \ell-1,$$

$$(3.25) \quad \lambda_0 + E\left\{\int_0^\infty exp(-\rho\tau)\lambda_k(t+\tau)d\tau\,|\,\mathcal{F}(t)\right\} = 0$$

and

$$(3.26) \quad \lambda_k(t) - \rho E\left\{\int_0^\infty exp(-\rho\tau)\lambda_k(t+\tau)d\tau\,|\,\mathcal{F}(t)\right\} = 0.$$

Although it is not possible to characterize the complete solution to this problem for general forms of g, a simple implication for services can be derived[21].

Notice that the solution for $\lambda_k(t)$, to (3.26) is a martingale, hence $E\{\lambda_k(t+\tau)|\mathcal{F}(t)\} = \lambda_k(t)$. Relations (3.24) and (3.25) then imply that $\lambda_j(t) = -\lambda_k(t)/\rho^{j+1}$ for $j=0,1,2,\ \ldots\ \ell-1$. As a result we have:

$$(3.27) \quad E\left\{-g_\ell^f*[g_\ell*y_0(t) + s^2(t) - b(t)]\,|\,\mathcal{F}(t)\right\} = \lambda_k(t)/\rho^\ell.$$

The left side of (3.27) gives the marginal utility of a unit of the consumption good at time t. Hence we have the result that the marginal utility of consumption is a martingale. This result is analogous to Hall's (1978) result. Because the marginal utility of consumption is a martingale we have:

$$(3.28) \quad g_\ell^f*[s(t+\tau) - b(t+\tau)] = g_\ell^f*[s(t) - b(t)] + u(t+\tau),\ \tau>0$$

where I have substituted in the fact that $g_\ell*y_0(t)+s^2(t)=s(t)$ and where $E\{u(t+\tau)|\mathcal{F}(t)\} = 0$. To allow for the inversion of the convolution in

(3.28), I impose the following condition on g:

Assumption 3: $1/\tilde{g}(\zeta)$ is analytic for $\{\zeta: \mathrm{Re}(\zeta) > \rho/2\}$. Further, for each real $\sigma' > \rho/2$ and $\zeta = \sigma + i\omega$ where $\sigma > \sigma'$, $|1/\tilde{g}(\zeta)| \leq |\mathcal{P}_K(\zeta)|$ for some polynomial $\mathcal{P}_K$ which depends on compact sets $K \subset [\sigma', \infty)$ where $\sigma \in K$.

Assumption 3 implies that there is a one-sided forward looking distribution, g_i^f such that

$$(3.29) \qquad g_i^f * g_\ell^f = \Delta$$

where Δ is the dirac delta function. Applying g_i^f to (3.28) yields:

$$(3.30) \qquad s(t+\tau) - b(t+\tau) = s(t) - b(t) + g_i^f * u(t+\tau).$$

Since g_i^f is forward looking, and $E\{u(t+\tau)|\mathcal{F}(t)\} = 0$, (3.30) implies that:

$$(3.31) \qquad E\{s(t+\tau) - b(t+\tau)|\mathcal{F}(t)\} = s(t) - b(t), \ \tau>0,$$

or that s-b is a continuous-time martingale. This result can be summarized in the following Lemma:

Lemma: If the mapping g given in (3.5) satisfies Assumption 1, 2 and 3, then the process $(s-b)$ chosen by the consumer is a continuous-time martingale.

I will denote the time derivative of $(s\text{-b})[t]$ as: $D(s\text{-b})[t] = D\xi(t)$.

I will assume that $E(D\xi(t)^2) = \sigma^2 dt$.

III.F. Implications for Time-Averaged Consumption

Assumption 3 implies that there is a distribution g^s such that:

$$(3.32) \quad g^s * g = \Delta.$$

In fact the Laplace transform of the distribution g^s is $1/\tilde{g}(\zeta)$. This fact will be useful in the following sections. Using (3.32), the convolution in (3.5) can be inverted to yield a mapping from services to consumption:

$$(3.33) \quad g^s * s^1 = c.$$

Differencing (3.33) between t and t+1 yields:

$$(3.34) \quad g^s * [s^1(t+1) - s^1(t)] = c(t+1) - c(t).$$

Since s is a martingale and $s^1 = s - s^2$, (3.34) implies the following representation for the first difference of consumption:

$$(3.35) \quad c(t+1) - c(t) = g^s * \int_{t-1}^{t} D\xi(\tau) + g^s * [b(t+1) - b(t)] - g^s * [s^2(t+1) - s^2(t)].$$

Although (3.5) characterizes the dynamics of consumption, it is not completely useful for empirical work because of the presence of initial conditions. However if the initial conditions die quickly enough then

asymptotically their effect on the consumption law of motion will be small. I will impose the following condition upon the behavior of $g^{s}*s^{2}(t)$:

Assumption 4: There exists an $\nu > 0$ such that $lim_{t\to\infty} exp(\nu t) g^{s}*s^{2}(t) = 0$.

Assumption 4 implies that the initial conditions filtered through g^{s} die geometrically. Note that Assumption 4 imposes conditions both on the initial conditions $s^{2}(t)$ and the distribution g.

Given that the initial conditions and g are such that Assumption 4 is satisfied, for large t, first-differences in the consumption process are given approximately given by:

$$(3.36) \quad c(t+1) - c(t) = g^{s}*\int_{t-1}^{t} D\xi(\tau) + g^{s}*[b(t+1) - b(t)]$$

The error term in the approximation due to the omission of the initial conditions is of smaller order than t.

In the special case of a time-separable model in which $c(t) = s(t)$ and where $b(t)$ is a constant, (3.36) gives the familiar result that consumption follows a (continuous-time) random walk. As in Section 2, I interpret the observed consumption as being averages of consumption expenditure over a unit of time so that observed consumption is given by:

$$(3.37) \quad \bar{c}(t) = \int_{t-1}^{t} c(\tau), \ t=0,1,2, \ldots .$$

By averaging (3.36) over a unit of time, we have the following theorem:

Theorem: If the mapping g given in (3.5) and s^2 satisfy Assumptions 1, 2, 3 and 4 and the consumer chooses consumption according to problem (MP) then the observed consumption process $\{\bar{c}(t): t = 1,2,3, \ldots\}$ satisfies:

$$(3.38) \qquad \bar{c}(t) - \bar{c}(t-1) = g^s * w(t) + g^s * \left(\int_{t-1}^{t} [b(\tau) - b(\tau-1)]d\tau\right) + o(t)$$

$$\text{where } w(t) = \int_{t-1}^{t}\int_{\tau-1}^{\tau} D\xi(r)d\tau$$
$$= \int_{t-1}^{t}(t-\tau)D\xi(\tau) + \int_{t-2}^{t-1}(\tau-t+2)D\xi(\tau).$$

Since the term o(t) in (3.38) is asymptotically negligible, it will be ignored throughout the rest of the paper. Notice that $w(t)$ is a time-averaged martingale and that the first difference in consumption is obtained by "filtering" $w(t) + [b(t) - b(t-1)]$ through g^s.

III.G. More Restrictions on g are Needed

Let b(t) be constant, then (3.38) implies a continuous-time moving average representation for $\bar{c}(t) - \bar{c}(t-1)$ of the form:

$$(3.39) \qquad \bar{c}(t) - \bar{c}(t-1) = g^c * D\xi(t).$$

where $g^c = h * g^s$, $h(t) = t$ for $t \in [0,1]$, $h(t) = 2 - t$ for $t \in [1,2]$ and $h(t) = 0$ otherwise. If a continuous record of $\bar{c}(t) - \bar{c}(t-1)$ were available, then the function g^c (and as a result g^s) could be exactly identified. However, given that we only have the discrete-time observations: $\{\bar{c}(t)-\bar{c}(t-1):t=0,1,\ldots\}$ what can we learn about the form of g^s?

With discrete-time data, the best we can do is to identify the

discrete-time moving average representation for time-averaged consumption differences:

$$(3.40) \qquad \bar{c}(t) - \bar{c}(t-1) = \sum_{j=0}^{\infty} G^c(j)\epsilon(t-j)$$

where $E(\epsilon(t)^2) = 1$ and $E(\epsilon(t)\epsilon(\tau)) = 0$ for $\tau \neq t$. We would like to be able to map from the sequence $\{G^c\}$ to the function g^c that describes the time-nonseparabilities. Of course without further restriction it will not be possible to identify completely the function g^c. However, note that (3.40) is exactly the relationship that arises in a discrete-time version of the model in which the decision interval of the consumer is set equal to the interval of the data. In the discrete-time model, the function $G(j)$ reflects the *discrete-time* time-nonseparabilities in preferences. Suppose now we use the economic interpretation of the preferences from the discrete-time setting given representation (3.40). Would this economic interpretation be far wrong?

To answer this question, we can use the analysis of Marcet (1986) which shows how the sequence $\{G^c(j)\}$ is constructed as a function of g^c. To construct this mapping, let A be the closure (in L^2) of the set of all finite linear combinations of the functions of $t \in R^1$: $g^c(t-1)$, $g^c(t-2)$, $g^c(t-3)$, ... Let $a = g^c - Proj(g^c|A)$ where $Proj$ denotes the L^2 projection operator. Note that $a(t)$ is in general a convolution of $g^c(\tau)$ at many values of τ. Marcet (1986) shows that $G^c(j)$ is given by:

$$(3.41) \qquad G^c(j) = \frac{\int_0^{\infty} g^c(t+j)a(t)dt}{\int_0^{\infty} a(t)^2 dt}$$

$$(3.42) \qquad = -\hat{a} * g^c(j) / \int_0^{\infty} a(t)^2 dt$$

$$(3.43) \qquad = -\hat{a}*h*g^{s}(j).$$

where $\hat{a}(t) = a(-t)$.

If $G^{c}(j)$ were an average of the function $g^{s}(t)$ for values of t near j, then we would expect the discrete-time interpretation to be reasonably good. However (3.43) implies that the $G^{c}(j)$ is an average of the function $g^{s}(t)$ for all values of t and $G^{c}(j)$ will, in general, lead to poor inferences about the structure of preferences.

For example, consider the model of exponential depreciation of the consumption good, as given by (3.8). In this example, the function g^{c} is:

$$(3.44) \qquad g^{c}(t) = \begin{cases} 1-\delta t \text{ if } t \in [0,1] \\ \delta t - (1+2\delta) \text{ if } t \in (1,2] \\ 0 \text{ otherwise} \end{cases} .$$

Further

$$(3.45) \qquad Proj(g^{c}|A)(t) = \begin{cases} \beta(1+\delta-\delta t) \text{ if } t\in(1,2] \\ 0 \text{ otherwise} \end{cases} ,$$

where $\beta = (\delta^{2}/6 - 1)/(1-\delta+\delta^{2}/3)$. Notice that if $\delta=-\sqrt{6}$ then $\beta = 0$ and $a = g^{c}$. Further in this case the function $G^{c}(j)$ is zero for j not equal to zero. Hence consumption differences have the representation:

$$(3.46) \qquad \bar{c}(t) - \bar{c}(t-1) = G^{c}(0)\epsilon(t).$$

The discrete-time interpretation of this is that preferences are time-separable which they are not.

Since the function G^{c} will not in general allow us to obtain a correct

economic interpretation of preferences, the plan for the rest of the paper is to focus upon several simple forms of g^s that summarize intuitive types of time-nonseparability in preferences. These restricted forms of g^s allow for the estimation of the *continuous-time* form of g^s. In the next sections, I study the dynamics introduced by these preferences and examine how they perform empirically.

IV. An Illustration: Habit Persistence

The assumptions about the mapping from consumption to services introduced so far imply that the process $(s-b)$ is a martingale supported by consumption and capital stock processes that are members of $L^2(\Omega^+,\mathcal{P},Pr^+)$. In this section, I examine the example of habit and show what restrictions Assumptions 1 through 4 place upon the habit model. I also examine whether the solution potentially violates economic restrictions (such as nonnegativity of the capital stock and consumption) that are not imposed in the solution method.

The first assumption to be verified is Assumption 1. Notice that the Fourier transform of g′ for the habit persistence model of (3.12) is given by:

$$(4.1) \qquad \hat{g}'(\omega) = 1 - \frac{\alpha(-\gamma)}{i\omega - \gamma + \epsilon} = \frac{i\omega - \gamma + \epsilon - \alpha(-\gamma)}{i\omega - \gamma + \epsilon},$$

and

$$(4.2) \qquad \hat{g}'(\omega)\hat{g}'(\omega)^* = \frac{\omega^2 + [\epsilon - \gamma - \alpha(-\gamma)]^2}{\omega^2 + (\epsilon - \gamma)^2}.$$

In this case $\hat{g}'(\omega)\hat{g}'(\omega)^*$ is uniformly bounded and the space of consumption

processes consists of those processes such that $c \in \mathcal{L}^2(\Omega^+, \mathcal{P}, Pr^+)$. The Laplace transform of g is given by:

$$(4.3) \qquad \tilde{g}(\zeta) = \frac{\zeta - (1 - \alpha)\gamma}{\zeta - \gamma}$$

and the Laplace transform of g^s is given by:

$$(4.4) \qquad \tilde{g}^s(\zeta) = 1/\tilde{g}(\zeta) = \frac{\zeta - \gamma}{\zeta - (1-\alpha)\gamma}.$$

Notice that $\tilde{g}(\zeta)$ satisfies Assumption 2 if $\gamma < \rho/2$ and that Assumption 3 requires $\gamma^*=(1-\alpha)\gamma < \rho/2$. These restrictions allow γ to be larger than zero so that the habit effects of past consumption could grow over time. Also if $\gamma < 0$, α can be larger that 1 so that the effects of the habit stock are quite important. Although these extreme parameter settings are consistent with the model of Section 3, it is important to check whether they lead to a violation of economic restrictions that are not imposed in the solution.

To examine this issue, suppose that there is perfect certainty[22] so that the service process is a constant, $\bar{s}$. Under the simplifying assumption that $s^2(t)=0$, consumption at time t is given by:

$$(4.5) \qquad c(t) = g^s * \bar{s}.$$

If Assumption 3 were strengthened such that $1/\tilde{g}(\zeta)$ is analytic in the half plane $\sigma' > 0$, then as t approaches ∞, $c(t)$ would approach the constant $[1/\tilde{g}(0)]\bar{s}$. Hence the restriction that $c(t)$ be nonnegative is the

restriction that $1/\bar{g}(0) > 0$. However for many examples, a strengthening of Assumption 3 is not necessary.

In the habit persistence model under perfect certainty, consumption at time t is given by:

$$(4.6) \qquad c(t) = \left\{exp(\gamma^{*}t) - (\gamma/\gamma^{*})[exp(\gamma^{*}t) - 1]\right\}\bar{s}.$$

Notice that if $\gamma^{*} < 0$, then c(t) tends to $(\gamma/\gamma^{*})\bar{s}$ which is positive only if $\gamma<0$. If $\gamma^{*} = 0$ (by setting $\alpha = 1$), then $c(t) = \bar{s}(1 - \gamma t)$ which is again positive only if $\gamma<0$. If $\gamma^{*} > 0$ then, for large t, c(t) is approximately $exp(\gamma^{*}t)(1-\gamma/\gamma^{*})\bar{s} = exp(\gamma^{*}t)[\alpha/(\alpha-1)]\bar{s}$. For this to be positive, α must be large that one[23] and $\gamma^{*}>0$ implies that $\gamma < 0$. Hence the economic restriction that consumption is positive requires that $\gamma < 0$ which is a strengthening of Assumption 2.

Notice however that Assumption 3 is satisfied as long as $\gamma^{*}<\rho/2$. This allows $\alpha \geq 1$. When $\alpha = 1$, consumption grows linearly and when $\alpha > 1$, consumption grows geometrically at the rate γ^{*}. Although the nonnegativity restriction on consumption is satisfied in this case, the restriction that the capital stock remains positive needs to be checked.

The capital stock at time t is given by:

$$(4.7) \qquad k(t) = \int_0^{\infty} exp(-\rho\tau)c(t+\tau)d\tau - \int_0^{\infty} exp(-\rho\tau)e(t+\tau)d\tau \ .$$

Suppose that the endowment is constant over time at the level $\bar{e}$, then:

$$(4.8) \qquad k(t) = \int_0^{\infty} exp(-\rho\tau)c(t+\tau)d\tau - \bar{e}/\rho \ .$$

Notice that if $\gamma^* > 0$ (with $\gamma<0$) then the capital stock also is positive and grows at the rate γ^*. If $\gamma^* = 0$, then the capital stock grows linearly over time. If $\gamma^* < 0$ then the capital stock is constant over time. As a result, once the restriction $\gamma<0$ is imposed, no further restrictions besides those in Assumptions 1, 2 and 3 are necessary to ensure that consumption and the capital stock have reasonable dynamics.

The last assumption of Section 3 is the restriction that the initial conditions die geometrically as required by Assumption 4. In the habit persistence example $s^2(t) = -\alpha exp(\gamma t)x(0)$ and

$$\begin{aligned} g^s * s^2(t) &= -\alpha x(0)(D-\gamma)\int_0^t exp(\gamma^* \tau)exp(\gamma(t-\tau))d\tau \\ &= -\alpha x(0)(D-\gamma)\frac{\{exp(\gamma^* t) - exp(-\gamma t)\}}{(\gamma^* - \gamma)} \\ &= \alpha x(0)exp(\gamma^* t) \end{aligned} \tag{4.9}$$

Assumption 4 requires γ^* to be less that zero or $x(0)=0$. This is an additional restriction needed for the empirical work of Sections 5 and 6, but it is not a restriction that is implied by the economics of the model.

V. Empirical Results with Seasonally Adjusted Data

In this section, I empirically examine some examples of the model described in Section 3, using the seasonally-adjusted consumption data discussed in Section 2. In this section I will maintain the assumption that $b(t)$ is a constant for all t. Under this assumption, I show that if the durable nature of consumption goods is modeled then the model performs much better than the time-additive model of Section 2. I also show that there is

dramatic evidence against the pure form of habit persistence as given in (3.12). The habit persistence model performs somewhat better if the durable nature of the goods is modeled first. However, the model provides little improvement in the fit of the data over the simple exponential depreciation model.

V.A. Exponential Depreciation

The nondurables plus services consumption series examined in Section 2 contains several components that should be thought of as being durable. For example, clothing and shoes are components of nondurable consumption. For this reason, it is of interest to relax the time-separability assumption of Section 2 and model the durable nature of consumption.

One simple model of durables is given by the nonseparabilities induced by the mapping (3.8) of Section 3. Before the analysis of Section 3 can be applied, the assumed regularity conditions on the mapping g must be checked. Consider first $\hat{g}'(\omega)\hat{g}'(\omega)^*$:

$$(5.1) \qquad \hat{g}'(\omega)\hat{g}'(\omega)^* = 1/[\omega^2 + (\epsilon-\delta)^2].$$

Notice that $(\epsilon^2+\omega^2)\hat{g}'(\omega)\hat{g}'(\omega)^*$ is essentially bounded. Hence the boundedness requirement of Assumption 1 is satisfied for a *negative* value of ℓ. As a result, the solution to the problem will involve a law of motion for consumption that is not necessarily a well defined stochastic processes. This can be seen by considering the Laplace transform of the mapping from services to consumption which is given by $1/\tilde{g}(\zeta) = \zeta - \delta$, which is the Laplace transform of the operator $(D-\delta)$. Hence consumption at time t is

given by:

$$(5.2) \qquad c(t) = Ds(t) - \delta s(t)$$
$$= D\xi(t) - \delta s(t).$$

Note that the goods process has a component that is the time derivative of a martingale and, as a result, the consumption process is not a real stochastic process but is a *generalized* stochastic process.[24] The consumer can tolerate very erratic consumption in this model, because the consumer "consumes" a weighted average of past consumption that is relatively smooth.

As it stands, this model does not directly fit within the framework of Section 3. However, a simple modification allows us to use the analysis of Section 3. Suppose instead of choosing consumption at time t, the consumer chooses *accumulated consumption* at time t, where accumulated consumption, c_a is given by:

$$(5.3) \qquad c_a(t) = \int_0^t c(\tau)d\tau.$$

Notice that if $c(t)$ satisfies (5.2), then $c^a(t)$ lies within $\mathcal{L}^2(\Omega^+,\mathcal{P},Pr^+)$. The mapping from consumption to services can be modified to be a mapping from accumulated consumption to services: $s = g_a * c_a(t)$. Here g_a is given by:

$$(5.4) \qquad g_a = \Delta + q$$

where q is the distribution given by:

$$(5.5) \qquad q(t) = \begin{cases} 0 \text{ if } t < 0 \\ \delta exp(\delta t)\, t \geq 0 \end{cases} .$$

Replacing consumption by accumulated consumption and the mapping g by g_a it can be shown that this structure satisfies the assumptions of section 2. The only remaining issue is how to interpret the capital accumulation problem. In this case, integrate (3.21) over time to yield:

$$(5.6) \qquad k(t) - k(0) = \rho\int_0^t k(\tau)d\tau + \int_0^t e(\tau)d\tau - c_a(t).$$

Let $k^*(t) = \int_0^t k(\tau)d\tau$ and $e^*(t) = \int_0^t e(\tau)d\tau + k(0)$, then (5.6) can be rewritten as:

$$(5.7) \qquad Dk^*(t) = \rho k^*(t) + e^*(t) - c_a(t).$$

This is the new capital accumulation constraint that corresponds to letting $c_a(t)$ be the control variable. With this modification[25] the exponential depreciation model is embedded in the model of Section 3 resulting in the law of motion for consumption given in (5.2).

Using the results of Section 3, we have:

$$(5.8) \qquad \bar{c}(t) - \bar{c}(t-1) = (D-\delta)w(t)$$
$$= \int_0^1(1-\delta\tau)D\xi(t-\tau) - \int_0^1[1+\delta(1-\tau)]D\xi(t-1-\tau)$$

As in the time-separable case, first differences in time-averaged consumption follow a first-order moving-average process. Hence all autocorrelations beyond the first order are zero. However, the first-order

autocorrelation value need <u>not</u> be 0.25 nor even positive. The first-order autocorrelation first differences of consumption is:

$$(5.9) \qquad R(1) = \frac{\delta^2/6 - 1}{2 + (2/3)\delta^2}$$

R(1) is plotted in Figure 1. Note that as δ goes to $-\infty$, the value of R(1) goes to 0.25 since, as δ is driven to $-\infty$, the consumption good becomes instantly perishable and preferences are time separable.

Notice that if $\delta = -\sqrt{6}$ (a half life of 0.28 periods for the consumption good) then R(1) = 0 and a discrete-time martingale model would fit the consumption data as noted in Section 3. Of course, this brings up a serious idenfication problem. If δ were equal to $-\sqrt{6}$ we would not be able to distinguish the continous-time model examined here and a discrete-time counterpart. We would however be able to reject the continuous-time time-separable model. If we tried to estimate the interval between decisions (as in Chrisitiano (1985)) using a time-separable specification, we would spuriously find that the decision interval is one period when, in fact, the true model is a countinuous-time one.

Notice also that if $|\delta| < \sqrt{6}$, R(1) is negative. This could explain the fact that monthly first-differences of consumption are negatively correlated even in the presence of time aggregation. Just as in (2.14), parameterize the moving average process given in (5.8) as $\bar{c}(t) - \bar{c}(t-1) = \theta_0\epsilon(t) + \theta_1\epsilon(t-1)$. This MA(1) can then be reparameterized in terms of θ_0 and δ, where δ ties down the parameter θ_1 through its affect on R(1) given in (5.10). Table 5.1 gives maximum-likelihood estimates of the MA(1) model using the seasonally adjusted monthly consumption data. The estimated value of δ is -1.36, implying that the consumption goods have a half-life of 0.51

months. If a quarter is used as the basic time interval, the value of δ implied by the *monthly* estimation is $3*(-1.358) = -4.07$.

Suppose now that we restrict the moving average representation for quarterly data by setting δ to -4.07. The results of this constrained estimation are given in Table 5.2. Also given in Table 5.2 are the results of a likelihood ratio test of this model against an unrestricted MA(1) model. Note that at the 10% level, neither data set would reject this restriction. Hence the monthly and quarterly first-order autocorrelation values can be reconciled with a model that adds a simple exponential depreciation story.

5.B. *Habit Persistence*

Pure Habit Persistence

Following Constantinides, services at t are given by (3.12). A useful way of deriving the mapping from services to consumption that I need, is to first assume that $s^2(t) = 0$, so that $s(t) = s^1(t)$. Taking the time derivative of (3.12) yields:

$$(5.10) \qquad Ds(t) = Dc(t) + \alpha\gamma^2\int_0^\infty exp(\gamma\tau)c(t-\tau)d\tau - \alpha\gamma c(t)$$

or

$$(5.11) \qquad Dc(t) = D\xi(t) - \alpha\gamma c(t) - \alpha\gamma^2\int_0^\infty exp(\gamma\tau)c(t-\tau)d\tau,$$

since $Ds(t) = D\xi(t)$. This is the same expression derived by Constantinides (1990). The fact that the change in consumption is linked to an average of past consumption induces a type of smoothness in the consumption process

Using the results of Section 3 the spectral density of observed

consumption differences can be derived. Figure 2 plots the first-order autocorrelation value for the first difference in time-averaged consumption implied by this spectral density. R(1) is plotted as a fuction of γ for values of γ from -0.1 to -3, where the parameter α was set such that $\alpha(-\gamma)$ equals 0.5. This was done so that as γ goes to $-\infty$, the model approaches a time-additive one. Note that the autocorrelation value starts out below 0.2, moves above 0.25 and then approaches 0.25 as the habit persistence is driven out of the model and consumption approaches a martingale process. Habit persistence induces a great deal of smoothness in the consumption process in the sense that consumption differences are positively correlated[26]. Figure 3 gives the second-order autocorrelation value for differences in consumption. The second-order autocorrelation is small for all values of γ and can be negative for small values of γ. As γ goes towards $-\infty$, this value goes to zero since the consumption process approaches a time-averaged white noise process.

Maximum likelihood estimation[27] of this habit-persistence model using the quarterly consumption series yielded parameter estimates of α that were not significantly different from zero. This occurs because the positive first-order autocorrelation in quarterly data can be easily fit by the time-additive model (see Section 2). For the monthly data, the model did not fit well relative to the exponential depreciation model, because of the observed negative first-order autocorrelation.

Habit Persistence with Exponential Depreciation

The nonseparability induced by (3.12) captures the notion that people may develop a level of acceptable consumption over time and introduces

complementarity in consumption over time. However, how is consumption defined? Given the degree of durability in the consumption goods that we observe, it is of interest to first define an *intermediate service* process, which will be denoted s^*, where $s^*(t)$ gives the level of *durable* services flowing from the stock of consumption goods at time t. Habit is then developed over the intermediate service process by feeding s^*, instead of c, into the model of habit persistence given in (3.12).

The intermediate service process is generated according to:

$$(5.15) \quad s^*(t) = \int_0^\infty exp(\delta r)c(t-r)dr, \ \delta < 0$$

Again, habit persistence is modeled using (3.12), except that habit is developed over s^* instead of consumption directly:

$$(5.16) \quad s^1(t) = s^*(t) - \alpha(-\gamma)\int_0^\infty exp(\gamma r)s^*(t-r)dr, \ \gamma < 0, \ 0 < \alpha < 1$$

The initial conditions are modified in a similar manner.

Figure 4 plots the implied first-order autocorrelation value for the first-difference in time-averaged consumption as a function of δ and for $\gamma =$ -1, and -2. α was set to $0.5(-\gamma)$. These plots display the same characteristics as were found in Figure 1, namely with slow rates of depreciation (δ close to zero), negative autocorrelation in consumption is possible and as δ tends to $-\infty$ the autocorrelation asymptotes to a positive value. However, adding habit persistence allows the autocorrelation value to be negative or zero for more substantial amounts of durability. With gamma = -1, the autocorrelation value is negative if $|\delta|$ is smaller than 1.6. This allows durability with a half life of 0.44 periods for an

autocorrelation of zero, which is much larger than the value of 0.13 found without the presence of habit persistence.

Maximum likelihood estimates of the habit persistence with local durability model for the two quarterly data sets yielded very marginal improvement in the likelihood function over the pure exponential depreciation model. As a result, I do not report the results here.

Table 5.3 reports results of the estimation of the habit persistence with exponential depreciation model using monthly data. Table 5.3 also reports the values of a likelihood ratio test of the exponential depreciation model nested within the habit persistence with exponential depreciation model. This was calculated using the log-likelihood values reported in Table 5.1. This is a test of $\alpha=0$. However, when $\alpha = 0$, the parameter γ is not identified and the regularity conditions for the likelihood ratio test break down. Davies (1977) has suggested a test in this situation where the test statistic is found by evaluating the likelihood ratio statistic at all possible values of γ and taking the supremum of the resulting values. The value of the test statistic reported in Table 5.3 is a lower bound for Davies's statistic and has a chi-square distribution with one degree of freedom (assuming that γ is known and set to the estimated value of Table 5.3). The P-Value using this distribution is 0.067, hence at the 5% level the addition of habit persistence does not help the fit of the model.

Table 5.4 reports half-life values for the exponential depreciation and habit-persistence effects. The half-life of the durability of the good is 0.656 months. Notice that this half-life value is higher than the value of 0.51 months found with exponential depreciation alone. The half-life of the habit stock is estimated to be relatively large, but it is not precisely

measured. It seems that the degree of habit persistence is difficult to measure with this data set.

The results of this section indicate that once time aggregation is accounted for, there is very weak evidence for habit persistence in the nondurables plus services consumption series. Certainly to allow for the negative first-order autocorrelation in consumption differences found at monthly frequencies requires the modeling of durability in the data. Although adding habit persistence to the durability model does not significantly improve the fit of the model, this does not imply that habit persistence should be unimportant in understanding the behavior of asset prices.[28]

VI. Empirical Results with Seasonally Unadjusted Data

As we saw in Section 2, the continuous-time time-additive model does not fit the seasonally unadjusted consumption data. In this section, I examine whether the addition of temporal dependencies in preferences helps the fit of the model. The model of preferences implies that consumption differences display seasonality that can be captured by seasonal dummies and a seasonal autoregression.

The first piece of the model for seasonals is formed by assuming that the bliss point follows a deterministic seasonal pattern. Seasonal bliss point movement has been used in a similar way by Miron (1986).

Before adding this feature to the model, an *aliasing* problem must be faced. Suppose that the discrete-time data follows an exact deterministic seasonal pattern that could be fit with seasonal dummies. To model this in continuous time would require a function that also has this exact seasonal

pattern at the observed frequencies. However, for this continuous-time model there are an infinite number of unobserved frequencies. The class of observationally equivalent continuous-time functions have seasonal patterns with at least the frequencies determined by the period of the observed discrete-time data. As a result, I will assume that the bliss point follows a seasonal pattern in continuous time with frequencies corresponding exactly to the seasonal frequencies of the observed discrete-time data.

Assume that the data is observed at quarterly intervals. The bliss point is then assumed to be governed by:

$$(6.1) \qquad b(t) = \sum_{j=1}^{2}(\phi_j cos(t\pi j/2)+\beta_j sin(t\pi j/2))$$

where ϕ_1, ϕ_2, β_1 and β_2 are constants. Note that the observations of b at the integers, $\{b(t): t = 0,1,2, \ldots\}$, could be exactly fit by four seasonal dummies.

To determine how this model affects the behavior of observed consumption differences, difference and average (6.1) over time, and use (3.38) to yield:

$$(6.2) \qquad \bar{c}(t) - \bar{c}(t-1) = g^s*w(t) + g^s*B(t)$$

where

$$(6.3) \qquad B(t) = \sum_{j=1}^{2}(\hat{\alpha}_j[sin(t\pi j/2) + sin((t-2)\pi j/2)] \\ -\hat{\beta}_j[cos(t\pi j/2 + cos((t-2)\pi j/2)].$$

and where $\hat{\alpha}_j = 2\alpha_j/\pi$ and $\hat{\beta}_j = 2\beta_j/\pi$. Note that B(t) observed at the

integers can be represented using seasonal dummies and that the sum of these dummies over the period of one year is zero. Also note that $g^{*}*B(t)$ has these same characteristics. Relation (6.2) implies that observed consumption differences have a component that has an exact seasonal pattern and a random component with properties that are influenced by the form of the nonseparabilities.

6.A. *Seasonal Habit Persistence and Exponential Depreciation*

In order to first capture the durable nature of the goods and to capture the first-order autocorrelation noted in Table 2.5, consider an intermediate service process s^{*} just as in Section 5:

$$(6.4) \qquad s^{*}(t) = \int_{0}^{\infty} exp(\delta\tau)c(t-\tau)d\tau, \ \delta < 0.$$

As in the habit persistence model, the consumer is assumed to compare the current level of s^{*} to an average of past intermediate services. In this case instead of letting the habit stock be a weighted average of all past consumption, assume that the habit stock is given by the level of the intermediate service process of exactly one year ago.[29] In other words $s(t)$ is given by:

$$(6.5) \qquad s(t) = s^{*}(t) - \alpha s^{*}(t-4)$$

where, again, the basic time period is assumed to be a quarter. The Laplace transform of the distribution g for this case is given by:

(6.6) $\tilde{g}(\varsigma) = [1 - \alpha exp(-4\varsigma)]/(\varsigma - \delta)$.

Since the Laplace transform of g^s is $1/\tilde{g}(\varsigma)$, differences of time-averaged consumption can be represented as:

$$(6.7) \qquad \bar{c}(t) - \bar{c}(t-1) = \frac{(D-\delta)\int_{t-1}^{t} D\xi(\tau)}{(1 - \gamma L^4)} + \frac{(D-\delta)B(t)}{(1 - \gamma L^4)}$$

or

$$(6.8) \qquad \bar{c}(t) - \bar{c}(t-1) = \frac{\int_0^1 (1-\delta\tau)D\xi(t-\tau) - \int_0^1 [1+\delta(1-\tau)]D\xi(t-1-\tau)}{(1 - \gamma L^4)} + \frac{(D-\delta)B(t)}{(1 - \gamma L^4)} .$$

The representation given in (6.8) implies that first differences of time-averaged consumption have two pieces. The first is a random piece which has a first-order moving-average structure with a seasonal first order autoregression. The second piece is a set of deterministic seasonal dummies. Notice that if the model of (6.8) is correct, then seasonal adjustment removes some of the dynamics of consumption that are due to the preferences of the consumer. Hence applying the model to seasonally adjusted data will lead to misleading inferences about the structure of preferences[30].

Table 6.1 gives estimates of the parameters[31] of (6.8) for U.S. quarterly nondurables plus services data. For comparison Table 6.2 gives estimates of the same model with α set to zero. This is a model of pure exponential depreciation with no seasonal effects modeled through the inclusion of habit persistence as in (6.5). First note that the restriction $\alpha = 0$ is rejected by the data at the 0.1% level (values for the

likelihood ratio test are given in Table 6.1). This confirms the findings in Table 2.5 that the inclusion of seasonal dummies does not completely remove all seasonal effects. Estimation of an unrestricted ARMA(4,1) model for the random piece of consumption differences (after trend and seasonal dummies removed) yields a likelihood value of 150.02. A likelihood ratio test of the model given in (6.7) against this alternative yields a P-value of 0.184. This indicates that the model given in (6.8) is doing a reasonable job in representing the n.s.a. consumption data.

Notice that the estimated value of δ in Table 6.2 is much closer to zero than the value found with seasonally adjusted data. The estimated value for δ using monthly data seasonally adjusted data (see Table 7) is -1.358, which converts to $3\times(-1.358) = -5.074$ on a quarterly basis, whereas the corresponding value using unadjusted data is -1.802. The implied half-life using unadjusted data is 0.38 quarters versus 0.17 quarters using adjusted data. Estimation of a model with just seasonal habit persistence ($\delta = -\infty$) results in a log-likelihood value of 140.61. Testing this restriction against the model with δ unrestricted, results in a P-value of 0.0002 for the likelihood ratio test. Hence there is evidence of an important role for durability in the unadjusted data even at quarterly frequencies that does not exist in seasonally adjusted data.

In sum, the seasonally unadjusted data at quarterly frequencies indicates that durability in the goods is important and there is evidence for *seasonal* habit persistence[32]. The model with time-separable preferences does a very poor job in fitting this Quarterly data set. In Section 2, the evidence against the time-additive model came in the form of negative first-order autocorrelation in monthly data. Unfortunately, the Department of Commerce does not publish monthly, seasonally-unadjusted data, which

makes the corresponding exercise difficult to perform here.

VII. Concluding Remarks

In this paper I have developed and empirically investigated a linear quadratic model in which the representative consumer has temporally dependent preferences. The model demonstrates that time nonseparabilities in preferences interact with time aggregation in several important ways.

First, the interaction between these two effects produces rich dynamics in the consumption series. The presense of time nonseparabilities in preferences is important because they help to explain some of the observed behavior of aggregate consumption of nondurables plus services that cannot be explained by temporal aggregation alone. The seasonally adjusted consumption series is consistent with a model in which the consumption good is durable. There is some weak evidence for habit persistence effects, but only if the durable nature of the consumption goods is modeled first.

Second, the presence of nonseparabilities and a continuous-time decision framework may make models which ignore the time aggregation problem perform reasonably well on some dimensions. This produces a serious identification problem, making it difficult to determine the correct decision interval to use in modeling. The identification problem is very severe, since time nonseparabilities may yield implications that resemble those implied by a time-separable specification. This occurs due to the simultaneous presence of time aggregation and time-nonseparable preferences.

Third, the presence of time nonseparabilities can produce endogenous seasonal behavior. As a result, if some account is to be taken of time aggregation, it should probably be done with seasonally unadjusted data. In

fact, as I have shown, inferences about the structure of preferences are quite different between seasonally unadjusted and adjusted data. In particular, the use of seasonally adjusted data understates the importance of durability in consumption goods and hides the presence of habit persistence at seasonal frequencies.

The model I examined in this paper has two important simplifying features: linearity and a constant interest rate. This was done so that the behavior of consumption from the model could be easily developed. Further work also needs to be done in examining nonlinear environments and asset pricing environments that account for time nonseparability in preferences and the time aggregation problem.

FOOTNOTES

[1]See for example: Flavin (1981), Deaton (1986), Campbell and Deaton (1987) and Hansen and Singleton (1982, 1983).

[2]See also Grossman, Melino and Shiller (1985), Hansen and Singleton (1990), Litzenberger and Ronn (1986), Naik and Ronn (1987) and Hall (1988) for examples of taking account of temporal aggregation in asset pricing contexts.

[3]The fact that the continuous-time time-additive model does not fit the monthly data was also noted by Ermini (1989).

[4]Several other constraints need to be imposed upon the problem to make it well-posed economically. First, the consumer must not be allowed to choose the solution $c(t) = b$ which would result in perverse behavior of the capital stock. For instance, in the case of a sufficiently small endowment process and small initial capital stock, this would require that the capital stock go to negative infinity at a geometric rate. To make the model interesting, some other constraint needs to be imposed to rule this out as a possibility. Also, in choosing consumption at time t, the consumer is constrained by the information available at time t. Formal imposition of these constraints is discussed in the next section.

[5]See Hansen (1987) and Sargent (1986) for an examples of how the martingale implication arises in a discrete-time social planning environment similar to the one here.

[6]We actually observe averages of trending consumption. I will present results assuming that we observe averages of detrended consumption. The difference can be accounted for in all of the calculations presented in this paper. However the difference is very small and would needlessly complicate the presentation.

[7]Here, and throughout the paper, I define all processes before period 0 to be zero. As a result, (2.10) is well defined even for t-0 and t-1.

[8]If a law of motion for the endowment process was posited then the model would potentially imply a restriction across the law of motion for the endowment and consumption. A problem with this approach is that the model is this paper is of a single consumption good and observations of *e* (such as aggregate labor income) should be thought of as being split between many consumption goods. Modeling preferences over many consumption goods is well beyond the scope of this paper. See Hansen, Roberds and Sargent (1989) for further discussion of these issues.

[9]Seasonally unadjusted data is available from 1946. The longer data set was not used because the simple trend model used in this paper could not account for what appears to be a dampening of the variance of the seasonal component of the series over the longer time period. Modeling the change in the seasonal behavior of the series is beyond the scope of this paper. The shorter time period was chosen since this problem does not occur over this period.

[10]The trend removed in this case was estimated using a (potentially) misspecified likelihood function in which consumption differences were assumed to be uncorrelated over time.

[11]Standard errors of the estimates of R(1) are given in brackets. The standard errors for R(1) were calculated using the Newey-West (1987) procedure with one year of lags.

[12]In Section 6, I further examine seasonally unadjusted data to try to account for these effects.

[13]I will continue to use $\bar{c}$ to denote observed consumption even though here I am not considering the effects of time aggregation over the month.

[14]From here on I will ignore the role of measurement error.

[15]Unlike most analyses of Gorman-Lancaster technologies, I ignore nonnegativity constraints.

[16]Discrete-time versions of this specification of preferences have been considered by many authors. See for example Eichenbaum and Hansen (1990), Eichenbaum, Hansen and Richard (1987) and Hansen (1987), and the references therein.

[17]$\mathcal{P}$ is generated by the stochastic processes adapted to $\mathcal{F}$ that are continuous from the left and have right limits. See Chung and Williams (1983) or Elliot (1982) for a complete description of the predictable sigma-algebra and predictable processes.

[18]The restriction $\delta < \rho/2$ will be explained below.

[19]Beltrami and Wohlers (1966) show that if $\tilde{g}(\xi)$ is analytic in the half plane $\{\xi : \mathrm{Re}(\xi) \geq 0\}$ then g is a one-sided distribution. Here I can weaken the requirement on the analyticity of $\tilde{g}(\xi)$ since I can allow for distributions that induce growth.

[20]See, for example, Beltrami and Wohlers (1966, p. 28).

[21]In Section 4 I examine the behavior of the capital stock for the habit persistence example.

[22]Under perfect certainty, the habit model is a linear quadratic version of the model considered by Ryder and Heal (1973).

[23]Assuming that $\alpha \geq 0$ so that we are considering a model of habit persistence.

[24]See, for example, Gel'fand and Vilenkin (1964) for a discussion of generalized stochastic processes.

[25]Hansen, Heaton and Sargent (1990) show that the optimization problem with this modification is well defined.

[26]Sundaresan (1989) and Detemple and Zapatero (1989) investigate whether habit persistence results in "smooth" consumption in the sense that habit persistence may lower the volatility of consumption for a given volatility in the endowment process.

[27]Throughout this section, the models were fit using the frequency domain approximation to the likelihood function suggested by Hannan (1970).

[28]For empirical investigations of this type see Constantinides and Ferson (1989) and Heaton (1990) for example. See also Abel (1990) for a discussion of a slightly different form of habit persistence in an asset pricing context.

[29]A more general model would be to set $s(t) = s^{*}(t) - [\alpha(1-\gamma)/(1-\gamma L^{4})]s^{*}(t-4)$ so that the habit stock is a weighted average of services one year ago, two years ago and so on. A likelihood ratio test of $\gamma = 0$ yields a test value of 0.26 (with γ allowed to be negative in the unrestricted estimation). When γ is set to zero, we get the model given in (6.5). Since the more complicated model provides very insignificant improvement in the likelihood function, it is not discussed.

[30]Heaton (1989) also examines models in which the durable good has a finite life. This induces peaks in the spectral density of consumption differences. If these peaks are close to seasonal frequencies, then seasonal adjustment will also remove dynamics that are induced by the nature of the consumption goods themselves.

[31]The estimated values of the seasonal dummies are omitted for simplicity.

[32]Osborn (1988) also suggests that there is an important role for habit persistence in fitting seasonally unadjusted data.

References

Abel, A.B. (1990). "Asset Prices Under Habit Formation and Catching Up with the Joneses," *A.E.R. Papers and Proceedings*, May: 38-42.

Beltrami, E.J. and M.R. Wohlers (1966). *Distributions and the Boundary Values of Analytic Functions*. New York: Academic Press.

Campbell, J. Y. (1987). "Does Saving Anticipate Declining Labor Income? An Alternative Test of the Permanent Income Hypothesis," *Econometrica*, 55: 1249-1273.

Campbell, J. Y. and A. Deaton (1987). "Why is Consumption so Smooth?" Manuscript, Princeton University and NBER.

Campbell, J. Y. and N. G. Mankiw (1987). "Permanent Income, Current Income, and Consumption." Manuscript, Princeton University.

Chrisitano, L. (1985). "A Method for Estimating the Timing Interval in a Linear Econometric Model, with an Application to Taylor's Model of Staggered Contracts," *Journal of Economic Dynamics and Control* 9: 363-404

Christiano, L., M. Eichenbaum, and D. Marshall (1989). "The Permanent Income Hypothesis Revisited." Manuscript.

Chung, K.L. and R.J. Williams (1983). *Introduction to Stochastic Integration*. Boston: Birkhäuser.

Constantinides, G. M. (1990). "Habit Formation: A Resolution of the Equity Premium Puzzle.", *Journal of Political Economy*, 98: 519-543.

Davies, R. B. (1977). "Hypothesis Testing When a Nuisance Parameter is Present only under the Alternative," *Biometrica*, 64: 247-54.

Deaton, A. (1986). "Life-Cycle Models of Consumption: Is the Evidence Consistent with the Theory?" NBER Working Paper No. 1910.

Detemple, J.B. and F. Zapatero (1989). "Optimal Consumption-Portfolio Policies with Habit Formation," manuscript, Graduate School of Business, Columbia University.

Dunn, K.B., and K. J. Singleton (1986). "Modeling the Term Structure of Interest Rates Under Nonseparable Utility and Durability of Goods," *Journal of Financial Economics*, 17: 27-55.

Eichenbaum, M. S., and L. P. Hansen (1990). "Estimating Models with Intertemporal Substitution Using Aggregate Time Series Data," *Journal of Business Statistics*, 8: 53-69.

Eichenbaum, M. S., L. P. Hansen and S.F. Richard (1987). "Aggregation, Durable Goods and Nonseparable Preferences in an Equilibrium Asset Pricing Model," Program in Quantitiative Analysis Working Paper 87-9, N.O.R.C., Chicago.

Elliot, R.J. (1982). *Stochastic Calculus and Applications*. New York: Springer-Verlag.

Ermini, L. (1989). "Some New Evidence on the Timing of Consumption Decisions and on Their Generating Process," *Review of Economics and Statistics*, 71: 643-650.

Flavin, M. A. (1981). "The Adjustment of Consumption to Changing Expectations about Future Income," *Journal of Political Economy*, 89: 974-1009.

Ferson, W. and C.R. Harvey (1987). "Seasonality in Consumption Based Asset Pricing: An Analysis of Linear Models." Manuscript, Duke University.

Ferson, W. and G. M. Constantinides (1989). "Habit Persistence and Durability in Aggregate Consumption: Empirical Tests," Manuscript, University of Chicago.

Gel'fand, I. M. and N. Y. Vilenkin (1964). *Generalized Functions*. New Nork: Academic Press.

Grossman, S. J., and G. Laroque (1990). "Asset Pricing and Optimal Portfolio Choice in the Presence of Illiquid Durable Consumption Goods," *Econometrica*, 58: 25-51/

Grossman, S. J., A. Melino, and R. Shiller (1987). "Estimating the Continuous Time Consumption Based Asset Pricing Model," *Journal of Business and Economic Statistics*, 5: 315-327.

Hall, R. (1978). "Stochastic Implications of the Life Cycle-Permanent Income Hypothesis: Theory and Evidence," *Journal of Political Economy*, 86: 971-987.

_____ (1988). "Intertemporal Substitution in Consumption," *Journal of Political Economy*, 96: 339-357.

Hannan, E. J. (1970). *Multiple Time Series*. New York: Wiley.

Hansen, L. P. (1987). "Calculating Asset Prices in Three Example Economies." In Truman F. Bewley (ed.) *Advances in Econometrics*, vol. 4. Cambridge: Cambridge University Press.

Hansen, L. P., J. C. Heaton, and T. J. Sargent (1990). "Faster Methods for Solving Continuous-Time Recursive Models of Dynamic Economies," manuscript, Hoover Institution.

Hansen, L. P., W. Roberds and T. J. Sargent (1989). "Time Series Implications of Present Value Budget Balance and of Martingale Models of Consumption or Taxes." Manuscript, University of Chicago.

Hansen, L. P., and K. J. Singleton (1982). "Generalized Instrumental Variable Estimation of Nonlinear Rational Expectations Models," *Econometrica*, 50: 1269-1286.

_____ (1983). "Stochastic Consumption, Risk Aversion, and the Temporal Behavior of Asset Returns," *Journal of Political Economy*, 91: 249-265.

_____ (1988). "Efficient Estimation of Linear Asset Pricing Models with Moving-Average Errors." Manuscript, University of Chicago.

Hayashi, F. (1982). "The Permanent Income Hypothesis: Estimation and Testing by Instrumental Variables," *Journal of Political Economy*, 90: 895-916.

Heaton, J.C. (1989). "The Interaction Between Time-Nonseparable Preferences and Time Aggregation," Ph.D. dissertation, University of Chicago.

Heaton, J.C. (1990). "An Empirical Investigation of Asset Pricing with Temporally Dependent Preference Specifications," manuscript, M.I.T.

Hindy, A. and C. Huang (1989). "On Intertemporal Preferences in Continuous Time II: The Case of Uncertainty," Working paper No. 2105-89, Sloan School of Management, MIT, March 1989.

Huang, C., and D. Kreps (1987). "On Intertemporal Preferences with a Continuous Time Dimension, I: The Case of Certainty." Manuscript.

Jones, L. (1983). "Flow of Services and the Purchase of Durable Goods when Resale and Rental Markets are Missing." Manuscript.

Litzenberger, R. H., and E. I. Ronn (1986). "A Utility-Based Model of Common Stock Price Movements," *Journal of Finance*, 41, No.1: 67-92.

Marcet, Albert (1987). "Temporal Aggregation of Economic Time Series." Manuscript, Carnegie Mellon University.

Mankiw, N. G. (1982). "Hall's Consumption Hypothesis and Durable Goods," *Journal of Monetary Economics*, 10: 417-425.

Mehra, R. and E.C. Prescott (1985). "The Equity Premium: A Puzzle," *Journal of Monetary Economics*, 15: 145-161.

Miron, J. A. (1986). "Seasonal Fluctuations and the Life Cycle-Permanent Income Model of Consumption," *Journal of Political Economy*, 94: 1258-1279.

Nelson, C. R. (1987). "A Reappraisal of Recent Tests of the Permanent Income Hypothesis," *Journal of Political Economy*, 95: 641-646.

Newey, W.K. and K.D. West (1987). "A Simple Positive Semi-Definite Heteroskedasticity and Autocorrelation Consistent Covariance Matrix Estimator," *Econometrica*, 55: 703-708.

Naik, V.T. and E.I. Ronn (1987). "The Impact of Time Aggregation and Sampling Interval on the Estimation of Relative Risk Aversion and the Ex Ante Real Interest Rate." Manuscript.

Novales, A. (1990). "Solving Nonlinear Rational Expectations Models: A Stochastic Equilibrium Model of Interest Rates," *Econometrica*, 58: 93-111.

Osborn, D. (1988). "Seasonality and Habit Persistence in a Life Cycle Model of Consumption," *Journal of Applied Econometrics*, 3: 255-266.

Ogaki, M. (1988). "Learning About Preferences from Time Trends," Ph.D. Dissertation, University of Chicago, 1988.

Pollack, R.A. (1970). "Habit Formation and Dynamic Demand Functions," *Journal of Political Economy*, 78: 745-763.

Ryder, H.E., Jr., and G.M. Heal (1973). "Optimal Growth with Intertemporally Dependent Preferences," *Review of Economic Studies*, 40: 1-31.

Sargent, T. J. (1987). *Dynamic Macroeconomic Theory*. Cambridge: Harvard University Press.

Sundaresan, S.M. (1989). "Intertemporally Dependent Preferences and the Volatitlity of Consumption and Wealth," *Review of Financial Studies*, 2: 73-89.

Working, H. (1960). "Note on the Correlation of First Differences of Averages in a Random Chain," *Econometrica*, 28: 916-918.

TABLE 2.1

ESTIMATES OF MA(1) MODEL FOR FIRST-DIFFERENCES
OF PER CAPITA, SEASONALLY-ADJUSTED QUARTERLY CONSUMPTION
OF NONDURABLES PLUS SERVICES.
FIRST-ORDER AUTOCORRELATION RESTRICTED TO 0.25

Parameter	Parameter Estimates[a]	
	52,1 to 86,4	59,1 to 86,4
μ ($\times 10^2$)	0.475 (0.052)	0.485 (0.060)
θ_0	0.224 (0.016)	0.253 (0.019)
Log-Likelihood	92.491	67.046

[a]Standard errors are in parentheses.

TABLE 2.2

ESTIMATES OF MA(1) MODEL FOR FIRST-DIFFERENCES
OF PER CAPITA, SEASONALLY-ADJUSTED QUARTERLY CONSUMPTION
OF NONDURABLES PLUS SERVICES. UNRESTRICTED ESTIMATION

Parameter	Parameter Estimates[a]	
	52,1 to 86,4	59,1 to 86,4
μ ($\times 10^2$)	0.474 (0.053)	0.487 (0.059)
θ_0	0.224 (0.016)	0.252 (0.019)
θ_1	0.062 (0.019)	0.060 (0.023)
Log-Likelihood	92.494	67.092

[a]Standard errors are in parentheses.

TABLE 2.3

LIKELIHOOD RATIO TESTS OF TIME-SEPARABLE MODEL AGAINST HIGHER ORDER MOVING AVERAGE MODELS.
SEASONALLY-ADJUSTED QUARTERLY CONSUMPTION OF NONDURABLES PLUS SERVICES

Order of Nesting Moving Average Model	Likelihood Ratio Value (P-Value)	
	52,1 to 86,4	59,1 to 86,4
2	0.091 (0.956)	0.293 (0.864)
3	5.579 (0.134)	7.326 (0.062)
4	8.711 (0.069)	11.882 (0.018)
5	13.703 (0.018)	16.928 (0.005)

TABLE 2.4

ESTIMATES OF MA(1) MODEL FOR FIRST-DIFFERENCES
OF PER CAPITA, SEASONALLY-ADJUSTED MONTHLY CONSUMPTION
OF NONDURABLES PLUS SERVICES, 59,1 TO 86,12

Parameter	Estimation with 0.25 Restriction on R(1)[a]	Unrestricted[a]
μ $(\times 10^2)$	0.156 (0.027)	0.164 (0.017)
θ_0	0.248 (0.015)	0.216 (0.010)
θ_1	—	-0.047 (0.011)
Log-Likelihood	211.33	254.54

[a]Standard Errors are in parentheses.

TABLE 2.5

AUTOCORRELATION VALUES FOR FIRST-DIFFERENCES
OF PER CAPITA, NOT SEASONALLY-ADJUSTED QUARTERLY CONSUMPTION
OF NONDURABLES PLUS SERVICES, 59,1 TO 86,4
TREND AND SEASONAL DUMMIES REMOVED

Order of Autocorelation	Autocorrelation[a]
1	-0.059 (0.087)
2	-0.027 (0.083)
3	0.167 (0.107)
4	0.327 (0.080)
5	-0.073 (0.086)

[a]Standard errors are in parentheses. These were calculated using one year of lags in the Newey-West (1987) procedure.

TABLE 5.1

ESTIMATES OF EXPONENTIAL DEPRECIATION MODEL
FOR FIRST-DIFFERENCES OF PER CAPITA, SEASONALLY-ADJUSTED
MONTHLY CONSUMPTION OF NONDURABLES PLUS SERVICES, 59,1 TO 86,12

Parameter	Estimated Value[a]
μ $(\times 10^2)$	0.164 (0.017)
θ_0	0.215 (0.010)
δ	-1.358 (0.161)

Log-Likelihood 254.54

[a]Standard errors are in parentheses

TABLE 5.2

ESTIMATES OF EXPONENTIAL DEPRECIATION MODEL WITH $\delta = 3\times(-1.36)$, QUARTERLY DATA. ALSO LIKELIHOOD RATIO TESTS AGAINST UNRESTRICTED MA(1).

Parameter	Parameter Estimates[a]	
	52,1 to 86,4	59,1 to 86,4
μ $(\times 10^2)$	0.478 (0.048)	0.492 (0.055)
θ_0	0.225 (0.015)	0.253 (0.019)
Log-Likelihood	91.14	66.48
L.R. against unrestricted MA(1)[b]	2.69 (0.101)	1.16 (0.281)

[a]Standard errors are in parentheses

[b]Probability values are in parentheses

TABLE 5.3

MAXIMUM LIKELIHOOD ESTIMATES OF PARAMETERS FOR HABIT PERSISTENCE WITH LOCAL DURABILITY MODEL, USING FIRST-DIFFERENCES OF PER CAPITA, SEASONALLY-ADJUSTED MONTHLY CONSUMPTION OF NONDURABLES PLUS SERVICES

Parameter	Parameter Estimates[a]
μ $(\times 10^2)$	0.156 (0.024)
σ	0.132 (0.010)
δ	-1.558 (0.743)
γ	-0.211 (0.414)
α	0.438 (0.308)
Log-Likelihood	256.22
Improvement in log-likelihood over pure exponential depreciation model (Table 4.1)[b]	3.36 (0.067)

[a]Standard errors are in parentheses

[b]Probability value using Chi-square distribution with one degree of freedom in parentheses.

TABLE 5.4

ESTIMATES OF HALF-LIVES IN MONTHS
FOR DURABILITY AND HABIT PERSISTENCE EFFECTS WITH MONTHLY DATA

Depreciation Parameter	Estimates[a]
δ	0.656 (0.104)
γ	3.293 (6.473)

[a]Standard errors in parentheses

TABLE 6.1

ESTIMATES OF EXPONENTIAL DEPRECIATION MODEL
WITH SEASONAL HABIT FORMATION
FOR FIRST-DIFFERENCES OF PER CAPITA, SEASONALLY-UNADJUSTED
QUARTERLY CONSUMPTION OF NONDURABLES PLUS SERVICES, 59,1 TO 86,4.
ESTIMATES OF SEASONAL DUMMIES OMITTED

Parameter	Estimated Value[a]
μ $(\times 10^2)$	0.408 (0.074)
θ_0	0.127 (0.010)
δ	-1.802 (0.414)
α	0.349 (0.087)

Log-Likelihood	147.60
Likelihood Ratio test of pure exponential depreciation model (see Table 13)[b]	14.57 (0.0001)

[a]Standard errors are in parentheses

[b]Probability values are in parentheses

TABLE 6.2

ESTIMATES OF EXPONENTIAL DEPRECIATION MODEL
FOR FIRST-DIFFERENCES OF PER CAPITA, SEASONALLY-UNADJUSTED
QUARTERLY CONSUMPTION OF NONDURABLES PLUS SERVICES, 59,1 TO 86,4
ESTIMATES OF SEASONAL DUMMIES OMITTED

Parameter	Estimated Value[a]
μ ($\times 10^2$)	0.371 (0.060)
θ_0	0.139 (0.011)
δ	-2.449 (0.927)

Log-Likelihood 140.32

[a]Standard errors are in parentheses

Figure 1

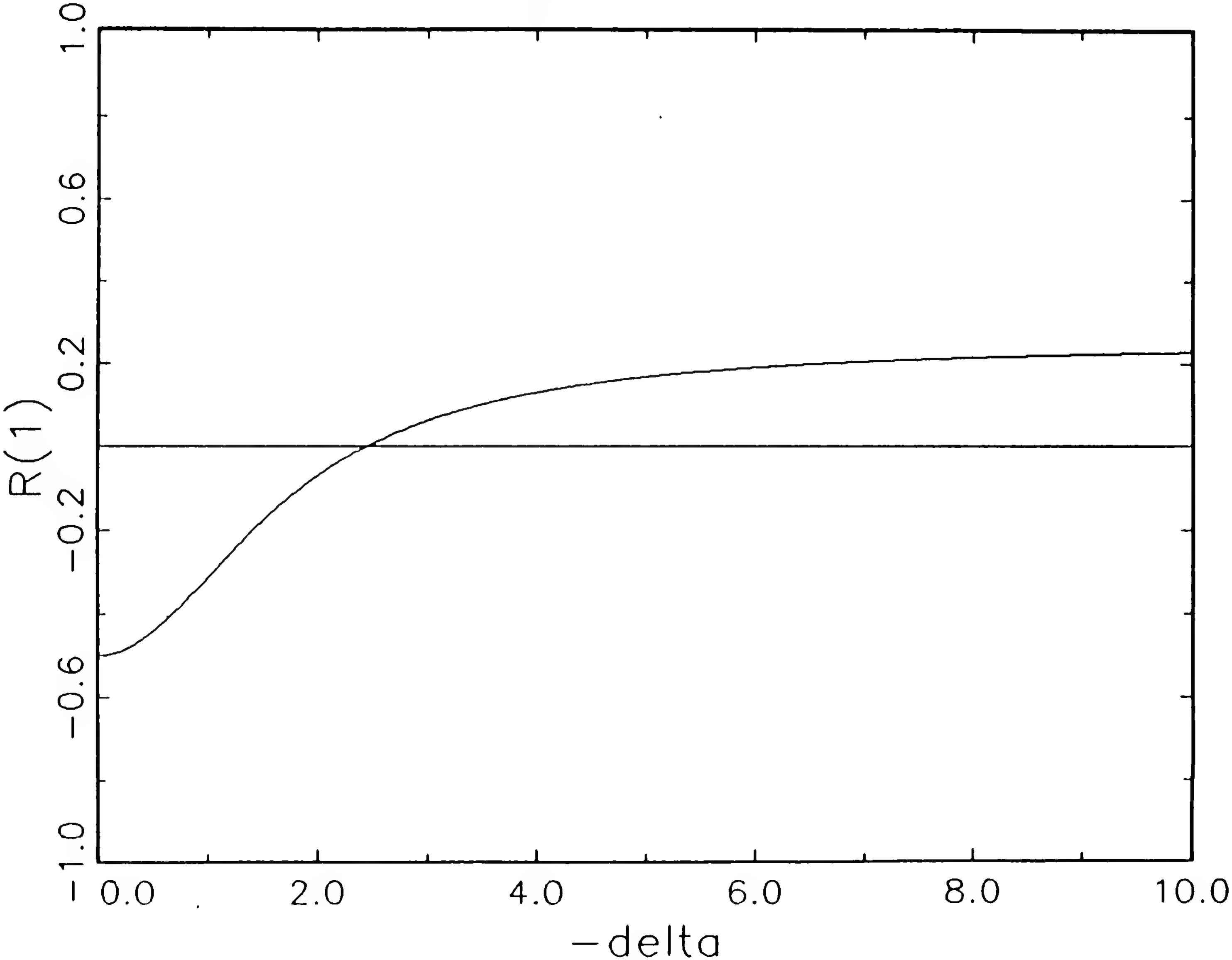

Figure 2

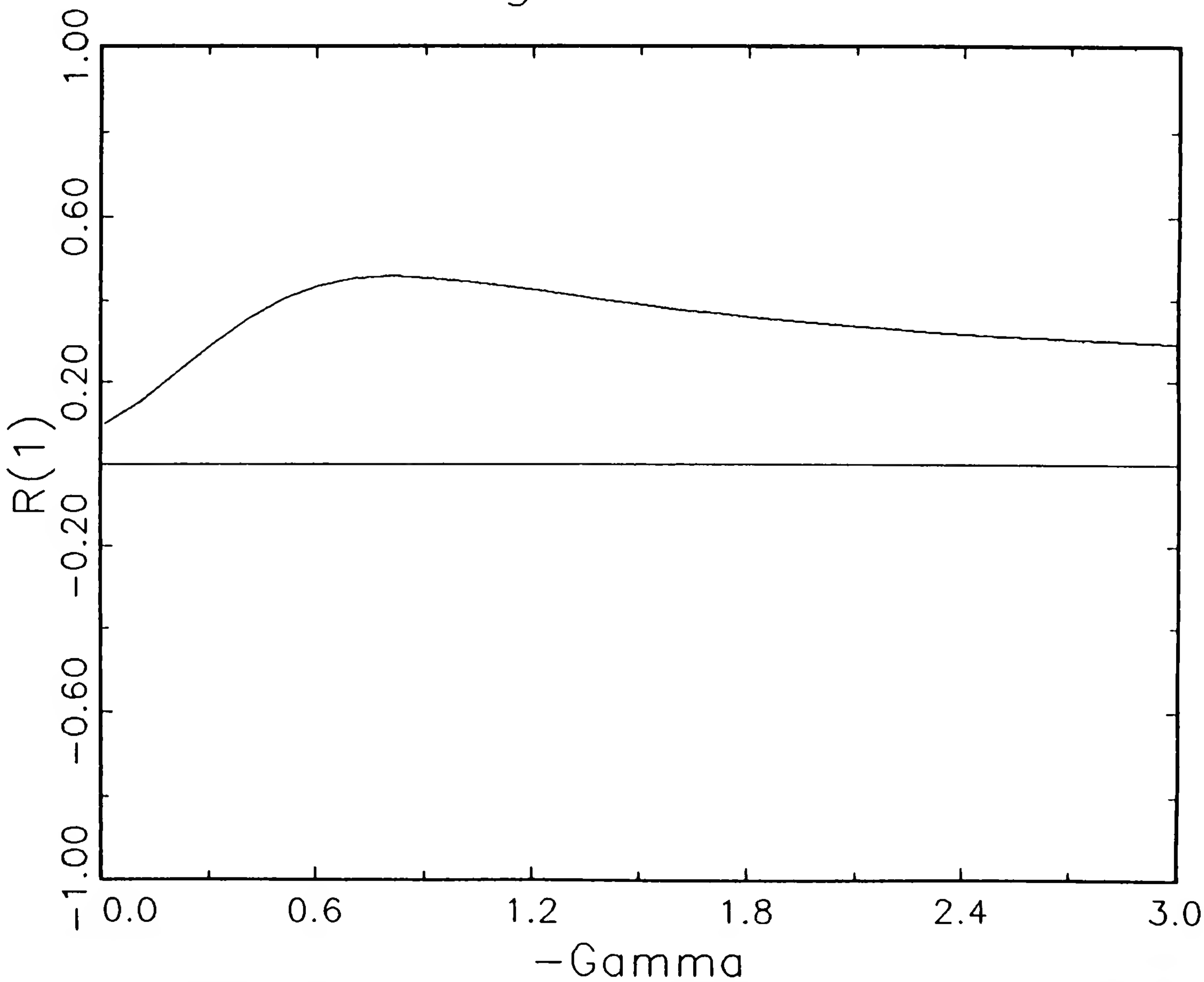

Figure 3

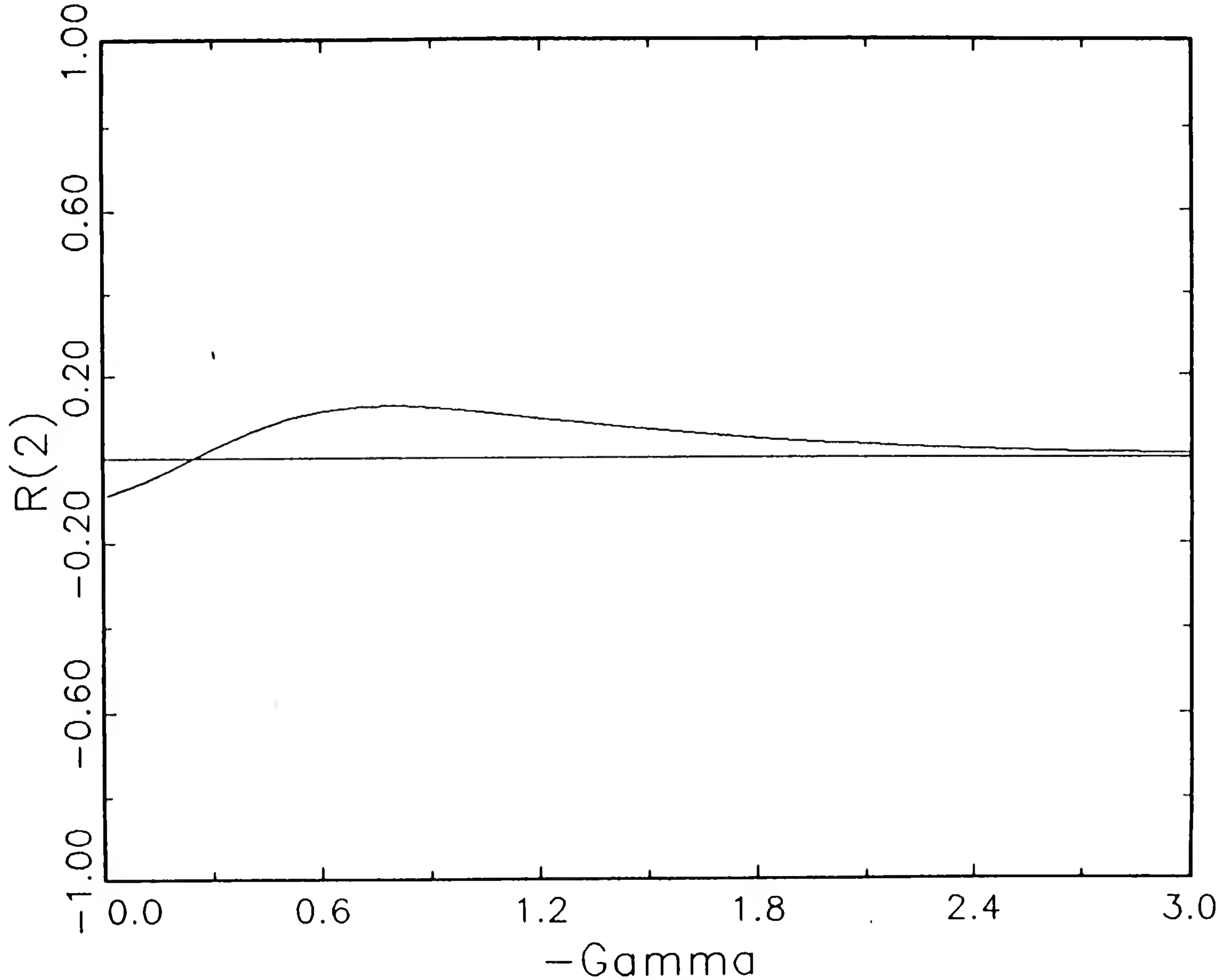

Figure 4